Plasmon-enhanced Faraday rotation in thin films

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We have analyzed analytically the Faraday rotation of an electromagnetic wave for a magnetoactive thin metallic film with a nanostructured surface profile. Periodic as well as random surface profiles were considered. The plasmon contribution to the Faraday angle was studied. For the periodic grating case, we have shown that the maximum rotation angle is achieved when the surface plasmon wave number coincides with one of the wave numbers of the inverse lattice. Enhancement of the Faraday angle at plasmonic band edges is predicted. In the case of a random surface profile, it is shown that the diffusion of surface magnetoplasmons gives a dominant contribution to the Faraday rotation. Comparison with experiments is carried out.

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I. INTRODUCTION

In recent years many experimental papers on enhanced Faraday rotation in systems with nanoscale inhomogeneities have appeared [1–7]. This increased research interest is largely motivated by the fact that the Faraday effect is widely used in optical isolators, phase modulators [8,9], spin dynamics [10], etc. Experiments found that the origin of the strong enhancement in the classical regime is intimately connected with the different plasmon [11] resonances. References [2,4,6] deal with three-dimensional (3D) random systems consisting of a solution with metallic nanoparticles embedded in it. In such systems, an enhancement of the Faraday rotation at frequencies close to the nanoparticle surface plasmon resonance frequency is observed in modest magnetic fields. Other experimental papers [1,3,7] were devoted to the plasmon-induced enhancement of Faraday rotation when an electromagnetic wave passes through a subwavelength thin metallic film with a nanostructured surface profile. The surface profile can be in the form of a periodic grating (as in the experiment [7] where inhomogeneity was created by the nanowires periodically placed on the surface) as well as random, as in the case of nanoparticles randomly embedded on the surface [1,3].

The theory, outlined in Ref. [12] for Faraday rotation in 3D disordered media, could correctly predict many of the peculiar features of the experiments [2,4,6]. According to Ref. [12] the Faraday rotation angle in a 3D disordered system is inversely proportional to the photon elastic mean free path, depends on the frequency, and has a minimum at the frequency of nanoparticle local plasmon resonance, due to a large scattering cross section.

However, most of the experiments on enhanced Faraday rotation are carried out with subwavelength thin metallic films that can be considered two-dimensional (2D) systems. A consistent theory of Faraday rotation in 2D disordered systems is absent.

In the present paper, we theoretically consider the Faraday rotation of light passing through a thin metallic film with a structured surface profile. Within a common approach we study both periodic and random surface profiles. We show that the plasmon scatterings on the inhomogeneities of the surface profile lead to rotation angle enhancement.

II. FORMULATION OF THE PROBLEM

Let a p-polarized wave impinge on the interface between two media (see Fig. 1). The incident wave magnetic field $\vec{H}$ is directed on $0y$. After passing the magneto-optical medium, it rotates in the plane xy. The plane of incidence of the wave vector is xz. The dielectric permittivity tensor of the system has the form

$$\varepsilon_{ij}(\vec{r}) = [\varepsilon_0(z) + \varepsilon_s(\vec{r})]\delta_{ij} - i e_{ijz} g, \tag{1}$$

where $\varepsilon_0(z) = 1$ if $z < 0$ and $z > L$ and $\varepsilon_0(z) = \varepsilon(\omega)$ if $L > z > 0$. The term ε_0 describes the smooth surface and $\varepsilon_s(\vec{r}) = (\varepsilon - 1)\delta(z)h(x,y)$ describes the surface roughness. Here $h(\vec{\rho})$ is the surface profile ($\vec{\rho}$ is a two-dimensional vector on the plane xy) that can be random as well as periodic and e_{ijk} is the antisymmetric tensor; g describes the magneto-optical properties of the medium, and we assume that the external magnetic field is directed on $0z$. The above geometry is more frequently used in the experiments. The Faraday rotation angle is determined as

$$\tan\theta = \frac{H_x(L)}{H_y(L)}, \tag{2}$$

where L is the thickness of the film. Assuming that the profile function $h(\vec{\rho})$ is smooth and neglecting its derivatives, from Maxwell equations one obtains a Helmholtz equation for the magnetic field,

$$\triangle H_i(\vec{r}) + \frac{\omega^2}{c^2}\varepsilon_{ij}(\vec{r})H_j(\vec{r}) = 0. \tag{3}$$

Substituting Eq. (1) into Eq. (3), it is easy to see that equations for right- and left-hand-polarized photons are separated:

$$\triangle H_\pm(\vec{r}) + k_0^2\varepsilon_\pm(\vec{r})H_\pm(\vec{r}) = 0, \tag{4}$$

where $k_0 = \omega/c$, $H_\pm = H_y \pm i H_x$, and $\varepsilon_\pm(\vec{r}) = \varepsilon_0 + \varepsilon_s \pm g$. We neglect the difference between left- and right-hand polarizations in ε_s because it is already proportional to a small

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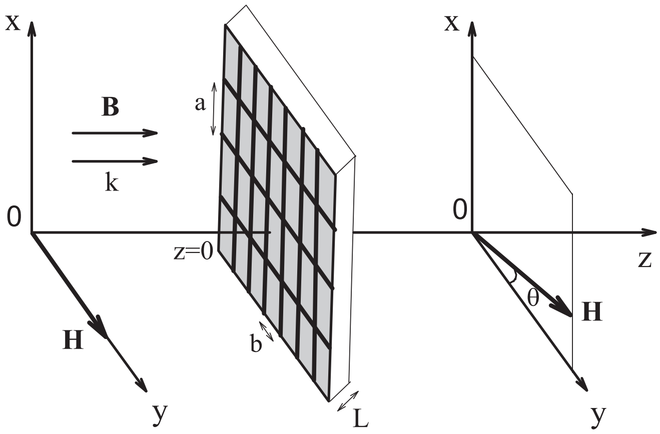

FIG. 1. Geometry of the problem. Incident wave is polarized on $0y$. The external magnetic field is directed on z. After passing through a thin film, the incoming beam is rotated by the Faraday angle θ.

parameter $h(\rho)$. The Faraday angle is determined through $H_\pm$ as

$$\tan\theta = -i\frac{H_+(L)-H_-(L)}{H_+(L)+H_-(L)}. \tag{5}$$

Because $H_{x,y}$ are continuous at $z=0$, the same is correct for $H_\pm$. From the Maxwell equations and continuity of $E_{x,y}$ follows the continuity of $(1/\varepsilon_\pm)\partial H_\pm/\partial z$ at $z=0$.

III. SMOOTH SURFACE

When the surface roughness is absent ($\varepsilon_s \equiv 0$), one can solve the Helmholtz equation, Eq. (3), for $\vec{H}$ with the above-mentioned boundary conditions and find

$$H_+^0(0) = 1 + r_+, \quad H_+^0(L) = t_+ e^{ik_0 L}, \tag{6}$$

where the reflection and transmission amplitudes are determined as follows (see, e.g., Ref. [13]):

$$r_+ = \frac{(\varepsilon_+ - 1)(e^{2ik_0L}-1)}{e^{2ik_0L}(1-\sqrt{\varepsilon_+})^2 - (1+\sqrt{\varepsilon_+})^2},$$
$$t_+ = e^{i(k_+-k_0)L}\left(1 + r_+\frac{1-\sqrt{\varepsilon_+}}{1+\sqrt{\varepsilon_+}}\right). \tag{7}$$

Here $k_+ = k_0\sqrt{\varepsilon_+}$ and analogous expressions for r_- and t_- can be written by substituting $+$ with $-$. Substituting Eqs. (6) and (7) into Eq. (5), for the thick films $k_0L \gg 1$ one finds the well-known result for the Faraday angle [14],

$$\theta_0 = \frac{gk_0L}{2\sqrt{\varepsilon}}. \tag{8}$$

In the thin-film limit $k_0L \ll 1$, similarly, we find

$$\theta_0 = \frac{gk_0L}{2}, \tag{9}$$

where we assume that $g \ll |\varepsilon|$. In an analogous manner one can find the polarization rotation angle for the reflected wave,

$$\tan\theta^R = \begin{cases} \frac{ig}{1-\varepsilon}, & k_0L \ll 1, \\ \frac{ig}{(1-\varepsilon)\sqrt{\varepsilon}}, & k_0L \gg 1. \end{cases} \tag{10}$$

Note that in both limits the rotation angle does not depend on L.

IV. SCATTERED FIELD

The solution of Eq. (4) consists of two contributions: one is caused by the smooth surface and the second one is caused by the scattering from the inhomogeneities $H_\pm(\vec{r}) = H_\pm^0(\vec{r}) + H_\pm^s(\vec{r})$, where the background field obeys a homogeneous equation

$$\triangle H_\pm^0(\vec{r}) + k_0^2\varepsilon_\pm H_\pm^0(\vec{r}) = 0. \tag{11}$$

The scattered field $H_\pm^s(\vec{r})$ is determined through the Green's function

$$H_\pm^s(\vec{r}) = -k_0^2\int d\vec{r}\,' G_\pm(\vec{r},\vec{r}\,')\varepsilon_s(\vec{r}\,')H_\pm^0(\vec{r}\,'), \tag{12}$$

where the Green's function obeys the equation

$$\triangle G_\pm(\vec{r},\vec{r}\,') + k_0^2(\varepsilon_\pm + \varepsilon_s(\vec{r}))G_\pm(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,'). \tag{13}$$

Below, we separately consider the case when the surface of a metal film has a periodic grating and when the surface profile is random.

A. Periodic grating

In this case, $h(\vec{\rho})$ is a two-dimensional periodic function. One can expand the profile function into discrete Fourier series

$$h(\vec{\rho}) = \sum_{n,m} h_{nm}e^{i\vec{K}_{nm}\vec{\rho}}, \tag{14}$$

where $\vec{K}_{nm} \equiv (2\pi n/a, 2\pi m/b)$ are two-dimensional discrete vectors on the inverse lattice; a,b are the profile periods in the x,y directions, respectively; and $n,m = 0, \pm 1, \pm 2, \ldots$. When one of the periods tends to infinity, one recovers a one-dimensional periodic profile, considered in the experiment in Ref. [7]. Substituting Eq. (14) into Eq. (12), taking its 2D Fourier transforms, and integrating over z using the explicit form of $\varepsilon_s(\vec{r})$, one finds

$$H_+^s(\vec{r}) = -k_0^2(\varepsilon-1)H_+(0)\sum_{nm} h(K_{nm})e^{i\vec{K}_{nm}\vec{\rho}}G_+(\vec{K}_{nm}|Z,0^+), \tag{15}$$

where $G_+(\vec{p}|z,z')$ is the two-dimensional Fourier transform:

$$G_+(\vec{\rho}-\vec{\rho}\,',z,z') = \int \frac{d\vec{p}}{(2\pi)^2}G_+(\vec{p}|z,z')e^{i\vec{p}(\vec{\rho}-\vec{\rho}\,')}. \tag{16}$$

It is worth noticing that the presence of the δ function in the expression of ε_s will lead to the different values of any physical quantity at $z=0$, while the integral over z is evaluated. To avoid the problem with discontinuous physical quantities at $z=0$ in our further calculations, we take their value at $z=0^+$. One has an analogous expression for the left-hand-polarized component. Note that the background field $H_+^0(\vec{r})$ which is the solution of the homogeneous Eq. (11) depends only on z. In order to obtain the Faraday rotation angle [see Eq. (5) and Fig. 1], we need to evaluate the coherent part of the scattered field, Eq. (15), that is the part with a wave vector directed on z:

$$H_+^{sc}(z) = -k_0^2(\varepsilon - 1)H_+(0)\sum_{nm} h(K_{nm})G_+(\vec{K}_{nm}|z,0^+). \tag{17}$$

As seen from Eq. (17), the scattered field includes a Green's function that has a plasmon pole which plays a crucial role in our study of the magneto-optical effects in 2D disordered systems. More precisely, when one of the wave numbers of the inverse lattice K_{nm} coincides with the plasmon wave number, the scattered field resonantly enhances (see below).

In order to analyze the Faraday angle, taking into account the scattered field, let us represent it in the form

$$\tan\theta = -i\,\frac{H_+^0(L) - H_-^0(L) + H_+^{sc}(L) - H_-^{sc}(L)}{H_+^0(L) + H_-^0(L) + H_+^{sc}(L) + H_-^{sc}(L)}. \tag{18}$$

For the thin films, $H_+^0(L) - H_-^0(L) \sim L$. At the resonance, $H_+^{sc}(L) - H_-^{sc}(L)$ can be essentially larger than $H_+^0(L) - H_-^0(L)$. At the same time, $H_+^{sc}(L) + H_-^{sc}(L)$ is proportional to the roughness height h/λ and is significantly smaller than $H_+^0(L) + H_-^0(L)$. The latter is proportional to unity provided that $L \to 0$. Correspondingly the Faraday angle will resonantly enhance provided that the resonance condition is fulfilled. Such an experimental enhancement of Faraday rotation was observed in a recent experiment [7].

It is worth noting that for the reflected wave the above-mentioned effect is absent due to the fact that the denominator and numerator of Eq. (18) at the resonance are of the same order; i.e., $H_+^{R0}(0) \sim r_+ \sim L \to 0$. For the thick films, $k_0L \gg 1$, the resonance effect is possible.

In order to estimate the plasmon contribution to the Faraday rotation angle for a thin film, we assume that the grating height is small, $h \ll \lambda$, and we substitute the Green's function in Eq. (17) by the bare one ($\varepsilon_s \equiv 0$).

B. Magnetoplasmon Green's function

For a given z', the Green's function has a sense of the magnetic field of a point source. Therefore, it satisfies the same boundary conditions as a magnetic field, namely continuity of G and $(1/\varepsilon_\pm)\partial G_\pm/\partial z$. Solving Eq. (13) for $\varepsilon_s \equiv 0$ with the above-mentioned boundary conditions at $z = 0$, one obtains

$$G_+^0(p|z,0^+) = \frac{-i\,e^{i\sqrt{k_+^2 - p^2}\,z}}{\sqrt{k_+^2 - p^2} + \varepsilon_+\sqrt{k_0^2 - p^2}}, \quad z > 0. \tag{19}$$

It is easy to find the Green's function for other values of z,z' also. However, for our purposes the above-mentioned one is enough. Note also that here we consider only the $z = 0$ plasmon contribution, believing that it is more important due to the roughness at $z = 0$ and not at $z = L$. One can also be convinced that the Green's function in Eq. (19) has a pole. To find the pole, we equate the denominator of Eq. (19) to 0. Getting free from the square roots near the pole values, the Green's function can be represented in the form

$$G_+^0(p|z,0^+) = \frac{a(K_{sp+})e^{i\sqrt{k_+^2 - K_{sp+}^2}\,z}}{K_{sp+}^2 - p^2 + i\,\frac{K_{sp+}}{l_{in+}}}, \tag{20}$$

where

$$K_{sp+}^2 = \frac{k_0^2\mathrm{Re}\varepsilon_+}{1 + \mathrm{Re}\varepsilon_+}, \quad l_{in+}^{-1} = \frac{k_0\mathrm{Im}\varepsilon_+}{\mathrm{Re}\varepsilon_+(1 + \mathrm{Re}\varepsilon_+)},$$

$$a(K_{sp+}) = -i\,\frac{\sqrt{k_+^2 - K_{sp+}^2} - \varepsilon_+\sqrt{k_0^2 - K_{sp+}^2}}{1 - \varepsilon_+^2}. \tag{21}$$

Here l_{in+} describes damping of the right-hand magnetoplasmon due to electromagnetic losses and we assume that $\mathrm{Re}\varepsilon_+ < -1$ and $|\mathrm{Re}\varepsilon_+| \gg \mathrm{Im}\varepsilon_+$. It follows from Eqs. (20) and (17) that the scattered field and corresponding Faraday angle will resonantly increase provided that one of the inverse lattice wave numbers K_{nm} coincides with the magnetoplasmon wave number K_{+sp}; see also Ref. [15]. Using Eqs. (20), (21), and (17) and keeping only the resonance term in the sum of Eq. (17), from Eq. (18) one has

$$\tan\theta_p \approx \frac{k_0^2h_0^2(\varepsilon - 1)}{2}\left[\frac{l_{in+}a(K_{sp+})}{K_{sp+}} - \frac{l_{in-}a(K_{sp-})}{K_{sp-}}\right], \tag{22}$$

where $h_0 \equiv h(K_{sp+})$ characterizes the height of periodic grating. To get the plasmon resonance contribution into the Faraday rotation angle in the periodic grating case, one has to substitute the parameters $K_{sp\pm}$ and $a(K_{sp\pm})$ from Eq. (21) into Eq. (22). The final result, in the limit $g \to 0$, reads as follows:

$$\tan\theta_p \approx -\frac{2igk_0h_0\sqrt{\varepsilon}}{\mathrm{Im}\varepsilon}. \tag{23}$$

Equation (23) with Eq. (34) (see below) represents the central results of this paper. The main difference of the plasmon resonance contribution $\tan\theta_p$ compared to a smooth surface metallic film contribution, Eq. (9), is that the former depends on the imaginary part of the dielectric permittivity $\mathrm{Im}\varepsilon$. Comparing Eqs. (9) and (23), we have

$$\frac{\mathrm{Re}\theta_p}{\mathrm{Re}\theta_0} \sim \frac{4h_0}{L}\frac{\sqrt{|\mathrm{Re}\varepsilon|}}{\mathrm{Im}\varepsilon}. \tag{24}$$

For nanoscale metallic films, usually $h_0 \sim L$. Taking into account that for noble metals in the optical region $|\mathrm{Re}\varepsilon| \gg \mathrm{Im}\varepsilon$, one has $\mathrm{Re}\theta_p \gg \mathrm{Re}\theta_0$.

To apply the obtained results to the experiment in Ref. [7], one can model the composite system consisting of a garnet substrate with a gold surface profile by metallic film with an effective dielectric permittivity tensor. The diagonal part of the latter is mainly determined by gold (at optical wavelengths its absolute value is much larger than that of garnet) and the nondiagonal part determined by the bismuth-substituted yttrium iron garnet value. Taking, at $\lambda = 963$ nm [7], $\mathrm{Re}\varepsilon \approx -40$, $\mathrm{Im}\varepsilon \approx 2.5$ [16], and $h_0 \sim L$, from Eq. (24) we obtain that the plasmon enhancement factor is of order 10. That agrees well with the experimental value of 8.9 [7]. For the profile periods $a = 495$ nm and $b = \infty$ the resonant number is $n = 2$. Note that the Faraday rotation angle for a smooth garnet film follows from Eqs. (8) and (9). Taking $g = 0.016$, $\varepsilon = 6.7$, $\lambda = 963$ nm, and $L = 150$ nm [7], one has $\theta_0 \sim 0.16^\circ$.

For large n,m one cannot separate out a single resonance term from the sum over the inverse lattice wave numbers, Eq. (17). In this case the summation can be replaced by integration ($\sum_K \to \frac{S}{(2\pi)^2}\int d\vec{K}$, where S is the area of the system). Carrying out the integration over $\vec{K}$ and making use of Eqs. (6), (7), and (18), one finds for the real part of the Faraday angle, in the limit $L \to 0$, the following expression $[G_\pm(\vec{\rho},\vec{\rho}) \equiv G_\pm(\vec{\rho},\vec{\rho},0^+,0^+)]$:

$$\mathrm{Re}\tan\theta_p = \frac{k_0^2(\varepsilon-1)h_0[\mathrm{Im}G_+(\vec{\rho},\vec{\rho}) - \mathrm{Im}G_-(\vec{\rho},\vec{\rho})]}{2}. \quad (25)$$

This is a general expression, independent of the surface periodic profile model. For simplicity we discuss only the constant harmonic grating case, i.e., $h(\vec{K}) = \text{const} = h_0$. The quantities $\mathrm{Im}G_\pm(\vec{\rho},\vec{\rho})$ are the local density of states of right- and left-hand-polarized magnetoplasmons. Because of translational invariance, they depend only on the difference of the arguments and therefore are independent of a local point ρ. Expanding $\mathrm{Im}G_\pm(\vec{\rho},\vec{\rho})$ on g, one gets that $\mathrm{Re}\tan\theta_p \sim g\partial\,\mathrm{Im}G/\partial\varepsilon$. Thus, the measurement of the rotation angle gives information on the density of states [17]. More as a consequence of the periodicity of $h(\vec{\rho})$, the plasmon spectrum consists of energetic bands and gaps. The above-mentioned derivative gets its maximal values at the edges of these bands. Similar behavior for the Faraday rotation was found in one-dimensional periodical systems [18,19].

C. Random surface profile

Now consider the case when the surface profile is random. We assume that $h(\vec{\rho})$ is a Gaussian distributed random function

$$\langle h(\vec{\rho})h(\vec{\rho}')\rangle = h^2\sigma^2\delta(\vec{\rho}-\vec{\rho}'), \quad \langle h(\vec{\rho})\rangle = 0, \quad (26)$$

where $\langle\cdots\rangle$ denotes the ensemble average and h and σ are the root-mean-square roughness and correlation length, respectively. In order to average the Faraday angle over the realizations of random roughness and to separate its real part, it is convenient to multiply the numerator and denominator of Eq. (5) by $H_+^*(L) + H_-^*(L)$ (see also [12]):

$$\langle\tan\theta\rangle \approx -i\frac{\langle(H_+(L)-H_-(L))(H_+^*(L)+H_-^*(L))\rangle}{\langle(H_+(L)+H_-(L))(H_+^*(L)+H_-^*(L))\rangle}. \quad (27)$$

Like in the periodical grating case, we decompose the magnetic field into background and scattered parts $H_\pm = H_\pm^0 + H_\pm^s$. In the denominator of Eq. (27) for small roughness $h \to 0$, one can keep only the terms containing background fields $H_\pm^0$. With respect to the numerator, one should keep only the terms containing the scattered fields because the terms associated with the background field are small for thin films: $H_+^0(L) - H_-^0(L) \sim L \to 0$. For the real part of the plasmon diffusional contribution to the Faraday angle, one finds from Eq. (27)

$$\mathrm{Re}\langle\tan\theta\rangle^D = -i\frac{\langle H_+^s H_-^{*s}\rangle - \langle H_-^s H_+^{*s}\rangle}{|H_+^0|^2 + \left|H_0^0\right|^2 + H_+^0 H_-^{*0} + H_-^0 H_+^{*0}}. \quad (28)$$

Recall that the scattered field is determined by Eq. (12) and that the terms $\langle|H_\pm^s|^2\rangle$ do not contribute to the real part of the Faraday angle because the denominator of Eq. (28) is real. To find the averages in Eq. (28) one needs the Green's function averaged over random roughness, taking into account the plasmon multiple scattering effects on the surface roughness (see Refs. [20,21]). Random roughness leads to a damping of the surface magnetoplasmon due to elastic scattering. The final answer for the magnetoplasmon Green's function, averaged over the randomness, reads

$$G_+(p|0^+,0^+) = \frac{a_+}{K_{sp+}^2 - p^2 + iK_{sp+}/l_+}, \quad (29)$$

where the magnetoplasmon elastic mean free path is determined as

$$l_+ = \frac{4K_{sp+}}{\beta a_+^2} \quad (30)$$

and $\beta = k_0^4(\varepsilon-1)^2h^2\sigma^2$. In Eq. (29) we neglect l_{in}^{-1} compared to l_+^{-1}. We take the contribution of the former into account in the diffusional propagator (see below). In the weak-scattering limit $K_{\pm sp}l_\pm \gg 1$, the main contribution to the average quantities in Eq. (28) gives the magnetoplasmon's diffusion. Using Eq. (12) one can represent the diffusional contribution in the form

$$\begin{aligned}\langle H_+^s(L)H_-^{*s}(L)\rangle^D &= \beta H_+^0(0)H_-^{*0}(0)P_{+-}(K=0)\\ &\quad\times\int\frac{d\vec{p}}{(2\pi)^2}G_+(\vec{p})G_-^*(-\vec{p})\\ &\quad\times\int\frac{d\vec{p}}{(2\pi)^2}G_+(\vec{p}|L,0)G_-^*(-\vec{p}|0,L),\end{aligned} \quad (31)$$

where $G_+(\vec{p}) \equiv G_+(\vec{p}|0^+,0^+)$, and P_{+-} is the magnetoplasmon diffusion propagator which is determined by ladder diagrams presented in Fig. 2 (see, for example, Refs. [22,23]). Summing the ladder diagrams in the limit $g \to 0$, one has

$$P_{+-}(K) = \frac{8\beta}{\frac{l}{l_{in}} + K^2l^2 - i\frac{gk_0l}{2(\varepsilon+1)^2}\sqrt{\frac{\varepsilon+1}{\varepsilon}}}. \quad (32)$$

Expression (32) was derived in the limits $Kl \ll 1$ and $\lambda \ll l \ll l_{in}$. The propagator $P_{-+}(K)$ is obtained from $P_{+-}(K)$ by changing the sign of g. Calculating the integrals in Eq. (31) in the limits $g \to 0$ and $L \to 0, k_0L\sqrt{|\varepsilon|} \ll 1$, we arrive at

$$\langle H_+^s(L)H_-^{*s}(L)\rangle^D = \frac{P_{+-}(K=0)}{\beta}. \quad (33)$$

Finally, using Eqs. (28), (32), and (33), for the diffusional contribution to the Faraday angle, we obtain

$$\langle\mathrm{Re}\tan\theta\rangle^D = \frac{2gK_{sp}l_{in}^2}{\varepsilon(\varepsilon+1)l}. \quad (34)$$

Note that the ratio l_{in}/l has is the average number of scatterings of the plasmon. A similar result for Faraday rotation in a

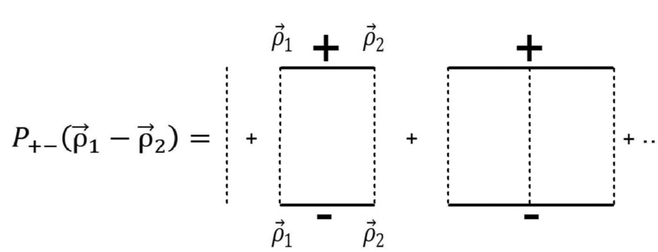

FIG. 2. Ladder diagrams. The top line is the right-hand-polarized magnetoplasmon averaged Green's function, the bottom line is the left-hand-polarized magnetoplasmon Green's function, and the dashed line is the random field correlation function $\beta\delta(\vec{\rho}_1-\vec{\rho}_2)$.

3D disordered medium is obtained in Ref. [12]. However, in 2D systems, Faraday rotation is more sensitive to the number of scatterings (square dependence against the linear in 3D case) as well as to the dielectric permittivity of thin film. Near the surface plasmon resonance $\varepsilon(\omega)+1=0$, the Faraday angle enhances because $\langle \mathrm{Re}\tan\theta\rangle^D \sim 1/\sqrt{|\varepsilon+1|}$. Comparing Eq. (9) with the flat surface contribution, Eq. (8), we have

$$\frac{\langle \mathrm{Re}\tan\theta\rangle^D}{\mathrm{Re}\tan\theta_0} \sim \frac{1}{\sqrt{\mathrm{Re}\varepsilon(\mathrm{Re}\varepsilon+1)^3}}\frac{l_{in}^2}{Ll}. \tag{35}$$

If the diffusion of the magnetoplasmon is realized on the film surface, i.e., the inequality $\lambda \ll l \ll l_{in}$ is met, then the condition $l_{in}^2 \gg Ll$ should hold. As a consequence, the diffusion contribution to the Faraday rotation angle can be the dominant one. Now let us make some numerical estimates to clarify whether or not the mentioned inequality takes place.

Assuming that the roughness is created by the nanoparticles randomly adsorbed on the surface, we have $h \sim \sigma \sim r$, where r is the radius of a nanoparticle. For gold nanoparticles with radius $r = 30$ nm at $\lambda = 600$ nm [3], $\mathrm{Re}\varepsilon = -7.8$, $\mathrm{Im}\varepsilon = 1.6$ [16], and $\beta = k_0^4(\varepsilon-1)^2h^2\sigma^2 \approx 0.75$. The surface plasmon wave number K_{sp} and the constant a in Eq. (21) are estimated as k_0 and $0.1k_0$, respectively. The surface plasmon elastic mean free path l is found from Eq. (30) to be approximately $l \sim 97\lambda$. If the losses are caused by the gold substrate, then the inelastic mean free path can be estimated using Eq. (21) and the above-mentioned numbers, $l_{in} \sim 5.3\lambda$. So the plasmon diffusion inequalities $\lambda \ll l \ll l_{in}$ are not realized under ordinary conditions.

However, close to the nanoparticle surface plasmon resonance the physical situation is completely different, because the expression in Eq. (30) for the determination of the plasmon elastic mean free path is not valid anymore. In this case the elastic mean free path can be estimated as $l = L/n_s\sigma$, where n_s is the surface concentration of nanoparticles and σ is the cross section of the interaction of the surface plasmon with the nanoparticle. When the surface plasmon wave number coincides with the nanoparticle surface plasmon resonance wave number, the plasmon elastic cross section resonantly enhances up to several orders compared to an ordinary situation; see, for example, Ref. [24]. Therefore, close to the resonance, the magnetoplasmon elastic mean free path becomes essentially smaller and the condition of its diffusion can be easily fulfilled.

V. SUMMARY

We have considered the Faraday rotation of light passing through a thin metallic film with nanostructured surface grating. In the periodic grating case, enhancement of the Faraday angle happens when the surface plasmon wave number coincides with one of the wave vectors of the inverse lattice, characterizing the grating periods on the surface. In the random surface profile case, the dominant contribution to the Faraday angle gives the diffusion of magnetoplasmons. If random roughness is created by the randomly embedded nanoparticles on the surface, then the maximum Faraday rotation angle of the transmitted wave is achieved when the surface plasmon wave number coincides with the nanoparticle surface plasmon resonance wave number. Experimental manifestations of the obtained results are discussed.

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