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The Escape Time of Electrons from Localised States

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The two components of the complex escape time $\tau^{\rm esc}(k) = \tau_1^{\rm esc}(k) - i\tau_2^{\rm esc}(k)$ for an electron from a localised state in a one-dimensional disordered system are shown to be connected by Kramers–Kronig integral relations. In the complex k plane, τ_1 and τ_2 form an elliptic contour. Component $\tau_2^{\rm esc}(k)$, in the case of an opaque barrier at an energy close to the bound level in the well, coincides with the lifetime expression.

Let us consider a potential shape that includes a well and one surrounding barrier (see Fig. 1). A hard wall condition at $x = -w$ reduces the problem to escape to only one open channel, i.e., transmission to the right.

To calculate the escape time $\tau^{\rm esc}(k)$ of an electron from a quantum well when boundary effects can be neglected, we closely followed Ref. [1] and introduced the following complex quantity ($k = \sqrt{E}$):

$$\tau^{\rm esc} = -i\,\frac{{\rm d}\ln t}{2k\,{\rm d}k}\,, \tag{1}$$

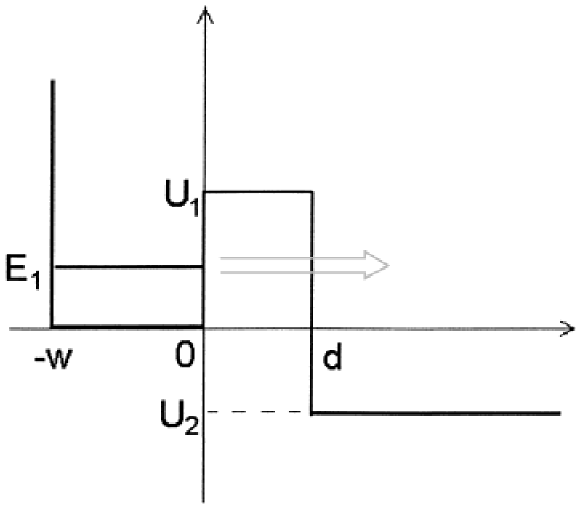

Fig. 1. Schematic representation of the simplified potential profile, with a hard wall condition at $-w$

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where $t = [T]^{1/2}\, e^{i\varphi}$ is the complex amplitude of transmission of the electron through only the right barrier taking into account the hard wall condition at $x = -w$.

Using standard methods of quantum mechanics and dropping all terms apart from the leading term in Eq. (1), we find for the Re and Im parts of the complex escape time the following asymptotic expressions near the resonance energy E_1 and in the limit of an opaque barrier:

$$\tau_1^{\rm esc}(k_1) = \frac{\xi_1(1+\gamma_1 w)\,(1+\beta_1^2)}{2k_1^2(1+\xi_1^2)} \exp{(2\gamma_1 d)}\,, \tag{2}$$

$$\tau_2^{\rm esc}(k_1) = \frac{(1+\gamma_1 w)\,(1+\beta_1^2)\,(1-\xi_1^2)}{4k_1^2(1+\xi_1^2)} \exp{(2\gamma_1 d)}\,, \tag{3}$$

where $\xi = \gamma/k_3$, $\gamma = \sqrt{U_1 - E_1}$, $k_3 = \sqrt{E_1 - U_2}$, $\beta = k/\gamma$, and w, d, U_1, U_2 are defined in Fig. 1.

At this point, let us examine the relationship between the two components we have defined above and the lifetime expression

$$\tau_{\rm LT}(k_1) = \frac{(1+\gamma_1 w)\,(1+\beta_1^2)\,(1+\xi_1^2)}{16k_1^2\xi_1} \exp{(2\gamma_1 d)}\,, \tag{4}$$

which follows from an approximate perturbative approach based on Bardeen's perturbation Hamiltonian [2].

Despite the similarity between Eqs. (2), (3) and (4), they only allow a qualitative comparison at the bound level E_1. Since components $\tau_1^{\rm esc}$ and $\tau_2^{\rm esc}$ show a sharp variation around this energy, a comparison with Eq. (4) as a function of energy is interesting in order to study their behaviour at the quasibound level, which is shifted with respect to E_1. Such comparison is shown in Fig. 2 for the following parameter values: $U_1 = 1$, $U_2 = -1$, $w = 3$, for two d values ($d = 5$ in Fig. 2a, and $d = 2$ in Fig. 2b). An opaque barrier has been chosen in Fig. 2a to ensure the accuracy of the lifetime expression, but in Fig. 2b the estimated relative error for the lifetime expression is about 16%. The vertical line corresponds to the ground energy level when the barrier width is infinite.

In Ref. [3], the logarithm of the complex transmission amplitude, i.e. $\ln t$, was shown to be an analytic function of the wave function k, in the entire complex k plane, and linear dispersion relations between the real and imaginary parts of $\ln t$ were found. Using these dispersion relations, and the fact that the complex escaping time (1) can be derived in terms of derivatives with respect to the potential barrier height, $V(x)$, rather than with respect to the particle energy [1], it is straightforward to show that the real and imaginary components of $\tau(k)$ verify the Kramers-Kronig integral relations

$$\tau_1^{\rm esc}(k) = -\frac{1}{\pi}\,\mathbf{P}\int_{-\infty}^{\infty} \frac{\tau_2^{\rm esc}(k')}{k'-k}\,{\rm d}k'\,, \tag{5}$$

$$\tau_2^{\rm esc}(k) = \frac{1}{\pi}\,\mathbf{P}\int_{-\infty}^{\infty} \frac{\tau_1^{\rm esc}(k')}{k'-k}\,{\rm d}k'\,, \tag{6}$$

where **P** signifies that the integral is taken in the sense of the principal value.

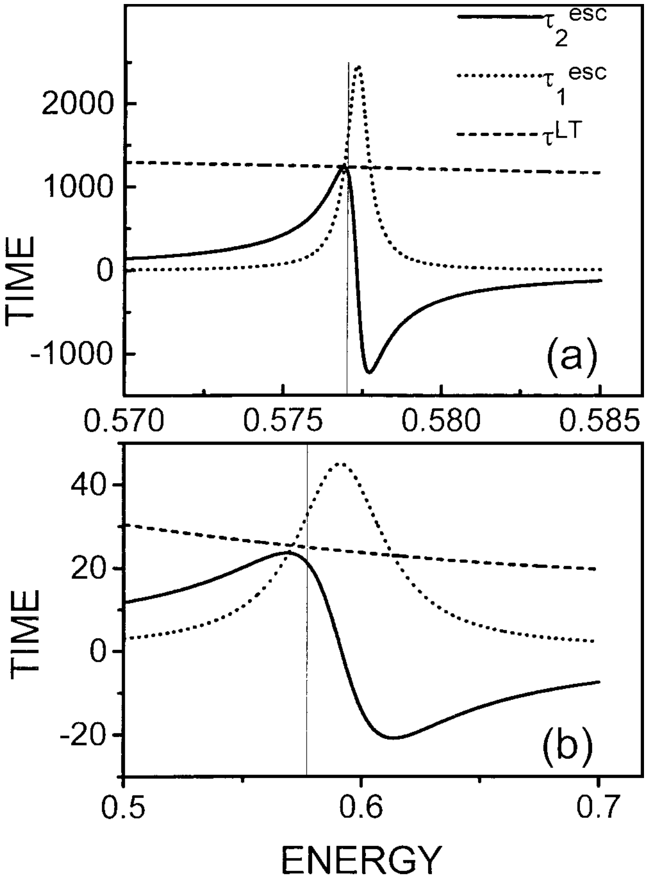

Fig. 2. The time components as a function of energy for the potential profile depicted in Fig. 1, with $U_1 = 1$, $U_2 = -1$, and $w = 3$, for two different d values: a) $d = 5$, and b) $d = 2$. The vertical line corresponds to the ground energy level when the barrier width is infinite

Let us represent, in the complex E plane, the Re and Im parts of the complex escape time (1) for the potential shape discussed in this paper. They are plotted one against the other in Fig. 3 and, as is seen, form an ellipse. This is what we expected, as the maxima of $\tau_1^{esc}(k)$ and $\tau_2^{esc}(k)$ are not the same. Nevertheless, there is a property of an ellipse that could be interesting: it is symmetric with respect to its main axis. Therefore, the maximum (and the minimum) of $\tau_2^{esc}(k)$ are found at the points where $\tau_1^{esc}(k)$ has a value of half its maximum. As this value is widely used to compute the lifetime (width of the $\tau_1^{esc}(k)$ peak at half height), this width must be exactly the difference in energies between the maximum and minimum of $\tau_2^{esc}(k)$. It is easy to check that we have the following condition:

$$(E_{\min} - E_{\max})\, \tau_2^{esc} = 1\,,$$

which confirms our previous conclusion concerning the fact that τ_2^{esc} coincides with the lifetime expression at an energy close to the bound level in the well.

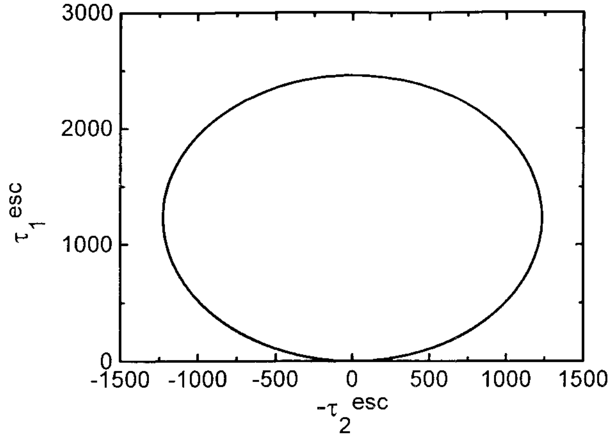

Fig. 3. Complex plane escape time diagram for the quantum well, used in this paper

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References

[1] J.A. LÓPEZ VILLANUEVA, and V. GASPARIAN, Phys. Lett. A **260**, 286 (1999).
[2] J. BARDEEN, Phys. Rev. Lett. **6**, 57 (1961).
[3] V.M. GASPARIAN, B.L. ALTSHULER, A.G. ARONOV, and Z.H. KASAMAIAN, Phys. Lett. A **132**, 201 (1988).
[4] V. GASPARIAN, G. SCHÖN, J. RUIZ, and M. ORTUÑO, Europ. Phys. J. B **9**, 283 (1999).
V. GASPARIAN, G. SCHÖN, J. RUIZ, and M. ORTUÑO, Ann. Phys. (Leipzig) **7**, N7-8, 756 (1998).