Z. Phys. B 100, 595–599 (1996)

ZEITSCHRIFT
FÜR PHYSIK B

Traversal time in periodically loaded waveguides

E. Cuevas[1], **V. Gasparian**[1,2], **M. Ortuño**[1], **J. Ruiz**[1]

[1] Departamento de Física, Universidad de Murcia, E-30080 Murcia, Spain
[2] Department of Physics, University of Yerevan, Yerevan, Armenia

Received: 7 March 1995/Revised version: 13 October 1995

Abstract. We derive general expressions for the transmission coefficient and the traversal time in waveguides periodically loaded with dielectric slabs. We apply the results to resonant tunneling between two regions below cut-off, with exponentially decaying modes. At the resonant frequencies, we found a relationship between the traversal time and the lifetime. We also study the photonic band gap in a periodic quarter wavelength structure. As in several experiments, we found superluminal velocities.

PACS: 41.20.Jb; 73.40.Gk; 84.40.Cb

1. Introduction

The question of the time required by an evanescent electromagnetic wave to cross a given region is a problem that has aroused much interest recently. This mode has a purely imaginary wavevector and we do not know the analogue of the classical velocity. The most direct method of calculating this time would be to follow the behavior of a wave packet and determine the delay due to the evanescent region, as was done by Martin and Landauer [1] for a waveguide with a dielectric slab. But this type of approach presents the problem that an emerging peak is not necessarily related to the incident peak in a causative way [2]. Also of physical significance is the time during which the mode interacts with the barrier, as measured by some physical clock which can detect the mode's presence within the barrier. As a clock we used the Faraday rotation of the polarization plane, produced by a weak magnetic field acting within the barrier region [3].

Evanescent waves are found in waveguides below their cut-off frequency. Enders and Nimtz [4, 5] found superluminal velocities in experiments on microwave evanescent mode transmission through undersized waveguides. The same was found by Ranfagni et al. using evanescent microwave pulses between two antenna horns [6]. Further confirmation of superluminal velocities was afforded by the measurements of single photon tunneling times, using a two-photon interferometer, on a periodic array of layers of material with an alternately high and low refractive index, which reflects most of the light in a given frequency range, due to interference effects [7].

Double-barrier potential structures present resonant tunneling, which has been studied for electrons since the early days of quantum mechanics [8–10]. Much of the physics of this phenomenon is also present in double undersized waveguides devices. Resonant tunneling for electromagnetic waves is easier to carry out than corresponding experiments on electrons [11]. Enders and Nimtz performed a series of microwaves experiments on double barrier systems and on the photonic band gap or non-propagating frequency region of periodic structures [4, 11]. These authors measured the transmission coefficient and the traversal time as a function of frequency and found an interesting structure in both magnitudes. In this paper we explain theoretically all their main results, using a previously developed method to calculate the traversal time [3, 12].

In the next section we summarize the method for calculating of the transmission coefficient and the tunneling time in layered systems, particularly, in waveguides. In Sect. 3 we apply these results to the experimental system used by Enders and Nimtz [5], i.e., a rectangular waveguide with two barriers. In Sect. 4 we analyze, for the same system, the relationship between lifetime and traversal time of a resonance. In Sect. 5, we obtain the transmission coefficient and the traversal time in the photonic band gap of a waveguide with a periodic arrangement of dielectric slabs. We finally discuss our theoretical predictions and reach some conclusions.

2. Tunneling time for layered waveguides

Let us consider a waveguide of uniform cross section filled with a material of dielectric constant ε_1 and with a different material, of dielectric constant ε_2 (we assume $\varepsilon_2 > \varepsilon_1$), arranged in a periodic structure. We assume that both materials are non-magnetic. Experiments concerning traversal times of evanescent modes are made in the frequency window where only the fundamental mode can propagate in the regions of the bigger dielectric constant since there is no propagating mode in the other regions. Alternatively, these experiments are made in the photonic band gap of periodic structures, where the non-propagating nature of the system is due to destructive interference of waves corresponding to different optical paths. Let us call γ the eigenvalue of the fundamental mode in the propagating regions (for a rectangular waveguide $\gamma = \pi/a$, where a is the larger edge of the rectangular section). The corresponding wavenumber is given by $k = \sqrt{\varepsilon_1 \omega^2/c^2 - \gamma^2}$ and the wavefunctions are of the form

$$A_l e^{ikz} + B_l e^{-ikz}, \tag{1}$$

where A_l and B_l are constant and the index l denotes the region. The temporal dependence has been omitted. In the evanescent regions, the inverse decaying length is equal to $\kappa = \sqrt{\gamma^2 - \varepsilon_2 \omega^2/c^2}$ and the wavefunctions are now of the form

$$A_l e^{kz} + B_l e^{-kz}. \tag{2}$$

Again A_l and B_l are constant and the index l denotes the region.

In this situation, where only one mode is relevant in each region, we can approximate the problem using a one-dimensional model. The boundary conditions for the electromagnetic field become equivalent to the continuity of the wavefunction and of its derivative (for TE modes; for TM modes in the continuity equation for the derivatives we have to multiply the derivative of the wavefunction by the inverse of the dielectric constant of each region) in analogy with the electron tunneling problem [1].

We can also obtain evanescent regions by diminishing the cross-section of the waveguide, the practice usually followed in experiments. The one mode approximation is not valid in this case because the higher order modes couple to the propagating modes, changing the reflection and transmission amplitudes. In the next section, we will explain how to modify the present results to include the effects of higher order modes.

Resolving the coefficients A_l and B_l, with the appropriate initial conditions, allows us, in principle, to calculate the transmission amplitude t. This can be done systematically by different methods and we choose the method based on the characteristic determinant [13], because of its very high efficiency. The details of this method can be found in [12].

We have used the Faraday rotation as a clock to measure the time spent by an electromagnetic wave in a given region. We found that, as for electrons, this time τ is a complex magnitude given by [3]

$$\tau = -i\left[\frac{\partial \ln t}{\partial \omega} - \frac{r}{\omega}\right] \equiv \tau_1 + i\tau_2 , \tag{3}$$

where r is the reflection amplitude. The real component of this time τ_1 corresponds to the delay time that is measured by keeping track of the peaks of the incident and the transmitted wavepackets. Experiments related to interaction times often measure the modulus of this complex time [14].

The characteristic determinant method allows us to calculate exactly the transmission amplitude and the traversal time of a periodic waveguide structure. Let us call d_2 the width of the evanescent regions and d_1 their separation. The periodicity of the system allows us to solve analytically the recurrence relationship for the characteristic determinant D_{N} (N is the number of interfaces)

$$D_{\mathrm{N}} = e^{ikd_1}\left\{\cos(N\beta a/2) - i\frac{\sin(N\beta a/2)}{\sin\beta a} \times \sqrt{\sin^2\beta a + \left[\frac{1}{2}\left(\frac{\kappa}{k} + \frac{k}{\kappa}\right)\sinh\kappa d_2\right]^2}\right\}, \tag{4}$$

where β plays the role of quasimomentum of the system, and is defined by

$$\cos\beta a = \cos kd_1 \cosh\kappa d_2 + \frac{1}{2}\left(\frac{\kappa}{k} - \frac{k}{\kappa}\right)\sin\kappa d_1 \sinh\kappa d_2 . \tag{5}$$

When the modulus of the RHS of (5) is greater than 1, β has to be taken as imaginary. This situation corresponds to a forbidden energy gap.

The previous expressions are also valid in those cases where two types of propagating regions exist, as in many experiments with periodic structures. We merely have to replace κ by ik_2, where $k_2 = \sqrt{\varepsilon_2\omega^2/c^2 - \gamma^2}$.

The transmission amplitude t is the inverse of the characteristic determinant, $t = 1/D_N$. From (4), we find that the inverse of the transmission coefficient is given by

$$|t|^{-2} = 1 + \left[\frac{1}{2}\left(\frac{\kappa}{k} + \frac{k}{\kappa}\right)\sinh\kappa d_2\right]^2 \frac{\sin^2(N\beta a/2)}{\sin^2\beta a} . \tag{6}$$

The term within brackets only depends on the properties of one barrier, while the quotient of the sine functions contains the information about the interference between different barriers. The transmission coefficient is equal to 1 when $\sin(N\beta a/2) = 0$ and β is different from 0. This condition occurs for

$$\beta a = \frac{2\pi n}{N} \qquad (n = 1, \cdots, N/2 - 1) , \tag{7}$$

and we say that it corresponds to a resonant frequency. For the reflection amplitude we have

$$r = \frac{i}{2D_N} e^{-ikd_1}\left(\frac{\kappa}{k} + \frac{k}{\kappa}\right)\sinh\kappa d_2 \frac{\sin(N\beta a/2)}{\sin\beta a} . \tag{8}$$

3. Transmission coefficient for a double barrier

We first analyze the behavior of the transmission coefficient as a function of frequency when there are two evanescent regions separated by a propagating one ($N = 4$). Our aim is to reproduce the experimental results by Enders and Nimtz [5]. As the evanescent regions used by them are undersized waveguides, it is necessary to modify our previous theory to include the effects of higher order modes. Even if these do not propagate, they couple to the propagating modes and change the transmission and reflection amplitudes.

The consideration of higher order modes at one interface between two waveguides of different cross section is equivalent to the inclusion of a shunt capacitive susceptance iB at the junction of the equivalent circuit. The value of this susceptance for an asymmetric E-plane step in the quasistatic approximation is [15, 16]

$$B = \frac{wa}{\pi c}\left[2\ln\frac{a^2-b^2}{4ab} + \left(\frac{a}{b}+\frac{b}{a}\right)\ln\frac{a+b}{a-b}\right], \tag{9}$$

where a and b are the width of the larger and smaller rectangular sections, respectively. We go beyond the quasistatic approximation by including the following correction term in the susceptance [15]:

$$\frac{2wa^3}{c\pi^3 b^2}\sum_{n=1}^{\infty}\frac{1}{n^3}\left[\left(1-\frac{w^2a^2}{c^2n^2\pi^2}\right)^{-1/2}-1\right]\sin^2\frac{n\pi b}{\mathrm{a}}. \tag{10}$$

The effect of the susceptance on the transmission and reflection amplitudes is, in second order in the variational solution [17], to replace the factor κ/k appearing in the expressions of the latter section by the factor $(\kappa/k) - iB$.

In Fig. 1 we plot the transmission coefficient versus frequency for the double barrier used in the experimental work of Enders and Nimtz [5]. We choose for the different parameters of the system the same values as in this reference. The three curves in Fig. 1 correspond to different values of the length of the propagating region, where the wave resonates between the two barriers. This length determines the number of resonances in the frequency window between the two cut-offs of the problem, corresponding to the propagating and the evanescent regions, respectively. The curves have been shifted, so the logarithmic vertical axis only represents relative values (one unit corresponds to one order of magnitude).

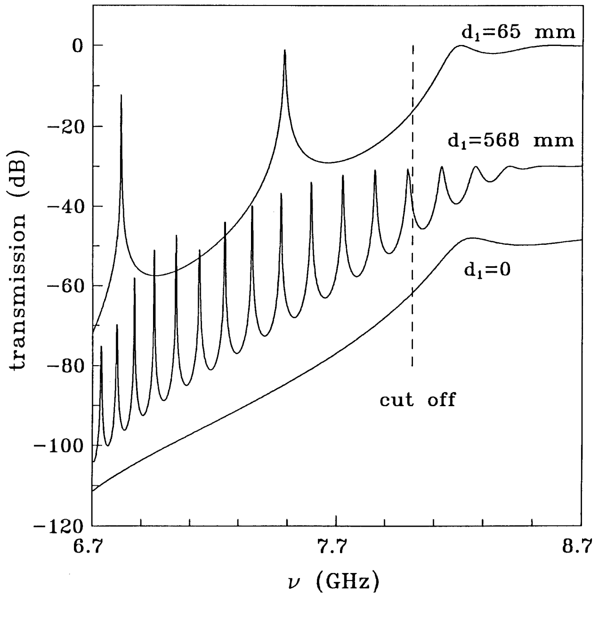

Fig. 1. Transmission coefficient versus frequency for a double barrier. The three curves correspond to different values of the separation between the two barriers. The curves have been shifted, so the logarithmic vertical axis only represents relative values. The vertical dotted line is the cut-off frequency of the barriers

Our results fit the experiments fairly well. If we would not consider the effects of higher order modes, there would be two small deviations between our theoretical results and the experimental ones: the peaks would be slightly shifted towards higher frequencies in the experimental results, and the slopes of the overall curves would be greater in the experiments. The inclusion of higher order modes in our treatment drastically reduces these two discrepancies and brings our results in line with the experimental ones.

The height of the peaks in Fig. 1 depends on the frequency resolution that we use. To simulate the finite resolution of the experimental set up, we convolute the ideal results with a gaussian distribution function with a given standard deviation, which is our only fitting parameter. In the graphs shown, we used a standard deviation of 6 MHz, of the order of the frequency width of the pulses used in the experiment, and which reproduces the same average height of the peak as the experiments [5].

4. Traversal time in a resonance

We calculate the traversal time τ of an electromagnetic wave through a double barrier structure by applying Eq. (3) to the previously obtained transmission and reflection amplitudes, once they have been convoluted with the corresponding gaussian distribution. In Fig. 2 we plot this traversal time (solid line) as a function of frequency together with the values measured by Enders and Nimtz (full circles) at the resonant frequencies [5]. The agreement is relatively good.

If we do not include the previously mentioned gaussian spreading, the traversal times at the resonances are much greater than the experimental values. This discrep-

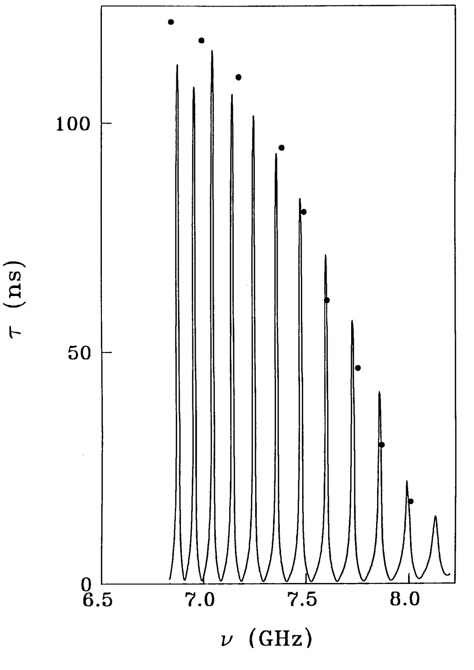

Fig. 2. Traversal time (solid line) and experimental resonant state decay times (full circles) versus frequency for a double barrier

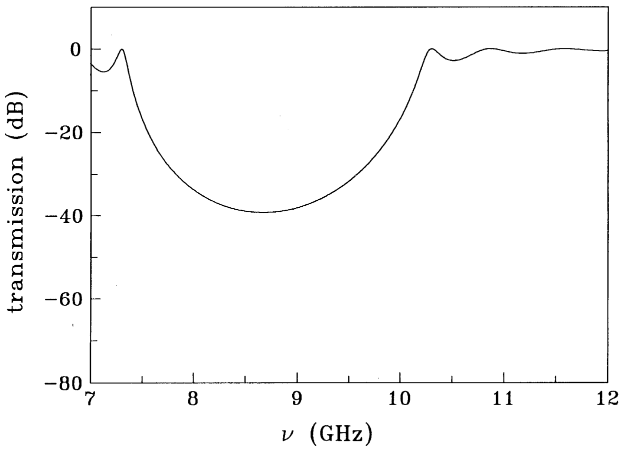

Fig. 3. Transmission coefficient of a periodic quarter wavelength structure as a function of frequency

ancy is especially important at low frequencies. Note, however, that the experimental measurement implicitly performs a similar spreading procedure due to the finite size of the wavepackets used.

The behavior of the traversal time at a resonance is fairly universal. The phase of the transmission amplitude changes by an angle of π at each resonance, as predicted for electrons by Friedel's sum rule and checked empirically by us for electromagnetic waves. Its frequency dependence can be fitted quite accurately by an arc tangent function. The time, proportional to the derivative of this phase is Lorentzian, with the same central frequency and width as the Lorentzian corresponding to the transmission coefficient. As the lifetime τ_l of the resonant state is the inverse of the width of the transmission coefficient at half maximum, we conclude that it must be equal to half the traversal time τ_{res} at the maximum of the resonant peak

$$\tau_l = \tfrac{1}{2}\tau_{res} \,. \qquad (11)$$

This result was previously obtained by Gasparian and Pollak [18] for the electronic case by considering the traversal time for an electron tunneling through a barrier with losses, i.e., with a decay time.

5. Results for periodically loaded waveguides

We have calculated the transmission coefficient and the traversal time for waveguides loaded with periodic arrangements of dielectric layers. We apply our results to the experimental system used by Nimtz et al. [11], with seven dielectric layers ($N = 14$ in our expressions). The thickness of each dielectric layer is $d_1 = 6$ mm and the separation between them is $d_2 = 12$ mm. These values correspond to quarter wavelength layers for the central frequency (8.7 GHz) and the refractive index ($n = 1.6$) of the dielectric layer used. In Fig. 3 we show the transmission coefficient as a function of frequency for this system. Our results agree with the experimental results. We find a first photonic band gap between approximately 7.5 and 10 GHz, the region where most of the incident wave is reflected.

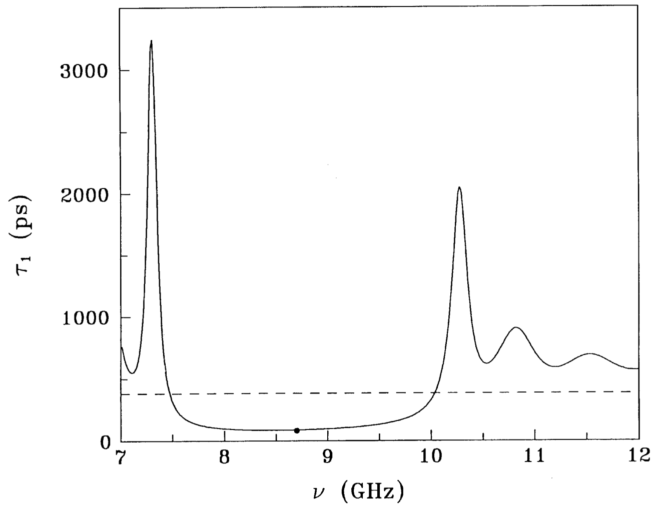

Fig. 4. Traversal time as a function of frequency for the same system as in Fig. 3. The dot corresponds to the experimental value of Nimtz et al. [11] and the horizontal line to the time needed to cross the sample at the vacuum speed of light

In Fig. 4 we present the traversal time as a function of frequency. The value of τ_l at 8.7 GHz is 88 ps, while the experimental result [11], represented by a dot in the figure, is 81 ps. In the non-propagating frequency window, where superluminal velocities are obtained, the traversal time is very small, in agreement with many previous theoretical and experimental results on exponentially decaying wave functions [2–7, 19]. The dotted horizontal line corresponds to the time needed to cross the sample at the vacuum speed of light.

6. Conclusions

The traversal times for exponentially decaying waves are very short and often correspond to superluminal velocities. This is so for both electrons and electromagnetic waves. Here, we have studied the properties of decaying electromagnetic waves in waveguides. We considered two cases, resonant tunneling between barriers below cut-off and photonic bands in periodic structures. Our results are in fairly good agreement with experiments on microwaves, which are a very good tool for studying the problem of superluminal velocities and their relation with information theory and the constraints imposed by relativity theory. An understanding of this problem will also be very helpful for the electronic tunneling case.

In this study we obtained a relationship between the lifetime of a resonance and the traversal time. It would be interesting to extend this result to a three-dimensional case.

We would like to thank the Dirección General de Investigación Científica y Técnica, project number PB 93/1125 and sabbatical support for VG (SAB93–0183), and the European Economic Community, contract number SSC*-CT90-0020, for financial support. V.G. also would like to acknowledge partial financial support from the ISF (grant no. MVL 000).

References

1. Martin, Th., Landauer, R.: Phys. Rev. A **45**, 2611 (1992)
2. Landauer, R.: Nature **365**, 692 (1993)
3. Gasparian, V., Ortuño, M., Ruiz, J., Cuevas, E.: Phys. Rev. Lett. **75**, 2312 (1995)
4. Enders, A., Nimtz, G.: Phys. Rev. E **48**, 632 (1993)
5. Enders, A., Nimtz, G.: Phys. Rev. B **47**, 9605 (1993)
6. Ranfagni, A., Fabeni, P., Pazzi, G.P., Mugnai, D.: Phys. Rev. E **48**, 1453 (1993)
7. Steinberg, A.M., Kwiat, P.G., Chiao, R.Y.: Phys. Rev. Lett. **71**, 708 (1993)
8. Sollner, T.C.L.G., Goodhue, W.D., Tannenwald, P.E., Parker, C.D., Peck, D.D.: Appl. Phys. Lett. **43**, 568 (1983)
9. Zohta, Y.: Solid State. Commun. **73**, 845 (1990)
10. Støvneng, J.A., Hauge, E.H.: Phys. Rev. B **44**, 13582 (1991)
11. Nimtz, G., Enders, A., Spieker, H.: J. Phys. I France **4**, 565 (1994)
12. Gasparian, V., Ortuño, M., Ruiz, J., Cuevas, E., Pollak, M.: Phys. Rev. B **51**, 6743 (1995)
13. Aronov, A.G., Gasparian, V., Gummich, U.: J. Phys.: Condens. Matter **3**, 3023 (1991)
14. Landauer, R., Martin, Th.: Rev. Mod. Phys. **66**, 217 (1994)
15. Collin, R.E.: Field Theory of Guided Waves. pp. 364–365. New York: Mc Graw-Hill 1960
16. Lewin, L.: Theory of Waveguides. p. 261. London: Newnes-Butterworths 1975
17. Rozzi, T., Mongiardo, M.: IEEE Trans. Microwave Theory Tech. **39**, 1279 (1991)
18. Gasparian, V., Pollak, M.: Phys. Rev. B **47**, 2038 (1993)
19. Enders, A., Nimtz, G.: J. Phys. I France **3**, 1089 (1993)