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BREWSTER ANOMALY AND TRANSMISSION OF LIGHT THROUGH ONE-DIMENSIONAL RANDOM LAYERED SYSTEM

A.G. Aronov

Leningrad Nuclear Physics Institute, Gatchina, Leningrad district, 188530, USSR

and

V.M. Gasparian

Department of Physics, Yerevan State University, ArmSSR, 375049, USSR

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The transmission of light through a one-dimensional random layered system is considered. The localization length depends on light polarization and goes to infinity approaching the Brewster angle for p-polarized light. The theory is in agreement with the results of Monte-Carlo simulation by Sipe *et al.* The angle of rotation of the polarization and the ellipticity of the light in a magnetic field are calculated in terms of the density of states and transmission.

AS IS KNOWN, electron states are localized in a one-dimensional disordered system. The average coefficient of transmission $\langle T \rangle$ at $L \to \infty$ (L is the length of system) is in the main exponentially decreasing in such a system. The electromagnetic wave when propagating through a random medium undergoes similar behaviour as well. At this, the exponential decrease of the coefficient $\langle T \rangle$ signifies that light is practically completely reflected from such a structure. However, as shown [1], the reverse, i.e., light practically completely passing through the one-dimensional structure, may be observed in specific case. The authors [1] have carried out Monte-Carlo simulation and have shown that for p-polarized light (electric vector $\mathbf{E}$ lies in the plane of incidence) at certain angles of incidence the depth of light penetration increases several orders as compared with the depth of penetration of the s-polarized wave (vector $\mathbf{E}$ is perpendicular to the plane of incidence). There, within the long-wave limit, the theoretical curve of the layered structure with dielectric constants ε_1 and ε_2, but with random thicknesses of layers, has been obtained for the depth of light penetration. Results of this approximation were only in qualitative agreement with Monte-Carlo simulation, though anomaly at the angle of incidence, equal to Brewster angle, was received.

In this paper, we shall analytically calculate the depth of light penetration into the disordered one-dimensional system, making use of the formula similar to the expression for the coefficient of transmission T, calculated in the paper [2].

Let us consider one-dimensional layered structure, each layer of which is characterized by dielectric constant ε_m and thickness a_m. An electromagnetic wave falls on such a system at the angle α, on the left. This problem is equivalent to the one-dimensional one due to homogeneity in the boundary plane (y, z). As a result, tangential components of wave vectors for all the waves are the same [3]. The coefficient of transmission T, precisely taking into account all multiplied reflections from all the boundaries, may be presented in the form

$$T_N = |D_N|^{-2} \tag{1}$$

where

$$D_N = D_N^0 \left\{ e^{2i\varphi_{1,N}} \prod_{m=1}^{N} (1 - r_{m-1,m}^2) \right\}^{-1/2}. \tag{1a}$$

Here N is the number of boundaries, $r_{m-1,m}$ are Frenel coefficients at light-reflecting from the separation boundaries of the two semi-infinite media: the wave from the semi-infinite medium with index $m-1$, falls on medium m, $\varphi_{1,N}$ is the phase set up by the wave when propagating along the whole sample.

$$\varphi_{1,N} = \sum_{m=1}^{N-1} k_m a_m; \quad k_m^2 = \varepsilon_m \frac{\omega^2}{c^2} - \mathbf{q}^2, \tag{1b}$$

$\mathbf{q}$ is the two-dimensional wave vector in the plane (axis x is directed perpendicularly to the surface of the random system), a_m is the thickness of m layer. Matrix elements $(\tilde{D}_N^{o})_{ml}$ are determined as

$$(\hat{D}_N^{o})_{ml} = \delta_{ml} + (1 - \delta_{ml}) r_{l,l-1} e^{i\varphi_{ml}}; \quad m \geqslant l$$
$$(\hat{D}_N^{0})_{ml} = \delta_{ml} + (1 - \delta_{ml}) r_{l-1,l} e^{i\varphi_{ml}}; \quad m \leqslant l \tag{2}$$

$$\varphi_{ml} = \varphi_{lm} = \sum_{i=\min(m,l)}^{-1+\max(m,l)} k_i a_i.$$

The determinant D_N^{o} satisfies the following recurrence relation

$$D_N^{o} = A_N D_{N-1}^{o} - B_N D_{N-2}^{o}$$

where

$$A_1 = 1; \quad D_{o}^{o} = 1; \quad D_{-1}^{o} = 0$$

$$A_N = 1 + \frac{r_{N-1,N}}{r_{N-2,N-1}} \exp(2i\varphi_{N-1,N}), \quad N > 1$$

$$B_N = (A_N - 1)(1 - r_{N-1,N-2}^2),$$

and $D_{N-1(N-2)}^{0}$ is determined by equation (2), in which the Nth [$(N-1)$th] line and column are absent. Expressions (1) and (2) are the generalization of the results obtained in Ref. [2] for layered structure.

Let us calculate the length of light localization in the model considered in [1]. The system is comprised of alternating layer thicknesses are distributed according to the law

$$P(a) = a_{o}^{-1} \exp(-a/a_{o}).$$

If Frenel coefficients for s and p polarization of light

$$r_s = (\cos\alpha - \sqrt{n^2 - \sin^2\alpha})(\cos\alpha + \sqrt{n^2 - \sin^2\alpha})^{-1} \tag{3}$$

$$r_p = (n^2\cos\alpha - \sqrt{n^2 - \sin^2\alpha}) \times (n^2\cos\alpha + \sqrt{n^2 - \sin^2\alpha})^{-1} \tag{4}$$

are small, then determinant D_N may be calcuated at arbitrary distribution of the thickness of layers $r_{m-1,m}^2 = r_{s,p}^2$

$$D_N^{-1} = e^{i\varphi_{1,N}} \prod_{m=1}^{N} (1 - r_{m-1,m}^2)^{1/2} \simeq \exp(i\varphi_{1,N} - \tfrac{1}{2} r_{s,p}^2) \tag{5}$$

correspondingly

$$T_{s,p} = e^{-Nr_{s,p}^2}. \tag{6}$$

Let us note that equations (5) and (6) are likewise true for the model, in which both layer thicknesses and dielectric constants ε_i fluctuate. In this case instead of $r_{s,p}^2$ the following expression must be involved

$$\langle r_{s,p}^2 \rangle = N^{-1} \sum_{m=1}^{N} r_{m-1,m}^2.$$

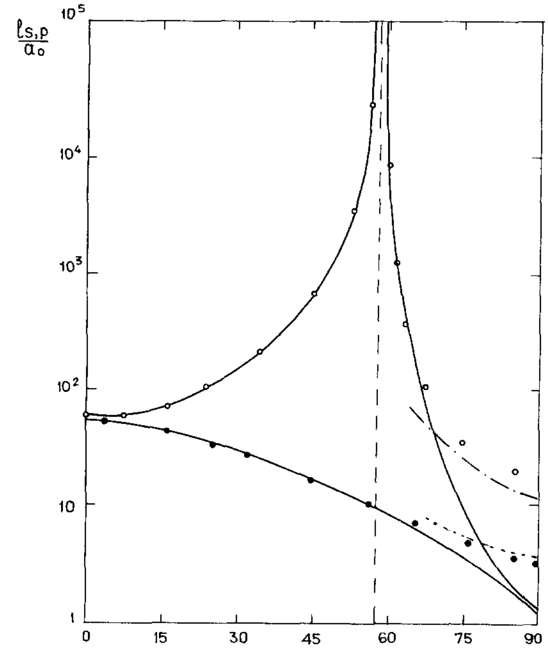

Fig. 1. The dependence of localization length vs the angle of light incidence at $\lambda = 5a_0$; $n^2 = 2.42$. Open circles are results of Monte-Carlo simulation for p-polarized light, and solid circles are for s-polarized light according to Ref. [1]. Solid and dashed lines are calculated from equation (7). Dash–dotted and dotted curves are calculated from equation (9).

Thus, localization length in this limiting case is

$$a_0/l_{s,p} = 2^{-1} r_{s,p}^2. \tag{7}$$

From expressions (3), (4) and (7) it follows that localization length l_p becomes infinite at Brewster angle tg $\alpha_B = n$. The comparison of equation (7) with the results of numerical experiment shows (see Fig. 1) that expression (7) describes well the experimental results up to angles $\alpha \lesssim 65°$ at $n^2 = 2.42$. It is necessary to note that one delocalized state appears with zeroes in all reflection coefficients on each boundary simultaneously and it is not connected with multipled reflection. At $\alpha > 65°$, reflection coefficients are not small and that is why multiplied transmission of light inside the layer is essential. In this case, all $r_{m-1,m}$ may be substituted for a unit, when calculating the determinant D_N^{o}. This results in

$$D_N = \frac{1}{2}\left(\frac{\sqrt{n^2 - \sin^2\alpha}}{n^2\cos\alpha}\right)^{N/2} \prod_{m=1}^{N/2} \sin k_1 a_m \sin k_2 a_{m+1} \tag{8}$$

where

$$k_1 = \frac{\omega}{c}\sqrt{\varepsilon_1}\cos\alpha, \qquad k_2 = \frac{\omega}{c}\sqrt{\varepsilon_2}\cos\varphi.$$

φ is the angle of refraction

$$\cos\varphi = n^{-1}\sqrt{n^2 - \sin^2\varphi}.$$

Localization length is determined by the expression

$$a_0/l_{s,p} = -N^{-1}\langle \ln T_N \rangle = N^{-1}\langle \ln |D_N|^2 \rangle,$$

$$N \to \infty.$$

If the thickness of layers is distributed according to Poisson law, the averaging should be carried out under the additional condition that the total thickness of all layers is equal $L = Na_0$,

$$\frac{2a_0}{l_{s,p}} = \ln\frac{\sqrt{n^2 - \sin^2\alpha}}{4\gamma^2 n^2 \cos\alpha} - \mathrm{Re}\left\{\psi\left(1 + \frac{i}{2k_1 a_0}\right) + \psi\left(1 + \frac{i}{2k_2 a_0}\right)\right\}, \quad (9)$$

where $\psi(x)$ is the di-gamma function, $\ln\gamma = C$ is the Euler number. At $k_i a_0 \ll 1$

$$\frac{a_0}{l_{s,p}} = \ln\frac{2\pi a_0}{\lambda\gamma n}\sqrt{n^2 - \sin^2\alpha} \quad (9a)$$

and at $k_i a_0 \gg 1$

$$\frac{2a_0}{l_{s,p}} = \ln\frac{\sqrt{n^2 - \sin^2\alpha}}{4n^2\cos\alpha}. \quad (9b)$$

The equation (9b) is not applied near the Brewster angle.

The comparison of equation (9) at large angles of incidence with the results of Monte-Carlo simulation is shown in Fig. 1. It is seen that the expression for the length of light localization (9) describes well experimental results at $\alpha > 70°$.

If one-dimensional random layered structure is placed in the external magnetic field $\mathbf{H}_0(\mathbf{H}_0 \parallel x)$, then each layer of such structure by dielectric tensor [3]

$$\varepsilon_{ik}^{(n)}(\mathbf{H}_0) = \varepsilon_{ki}^{(n)}(-\mathbf{H}_0)$$

$$\begin{pmatrix} \varepsilon^{(n)} & ig^{n}H_0 \\ -ig^{(n)}H_0 & \varepsilon^{(n)} \end{pmatrix}. \quad (10)$$

Condition $\varepsilon_{ik}^{(n)} = \varepsilon_{ki}^{(n)}$ signifies the absence of absorption in the medium, $g^{(n)}$ is the Faraday constant.

Let the linearly polarized planar wave normally fall on such a disordered structure. The direction of light propagation coincides with the magnetic field, i,e., with the axis x, and the direction of vector $\mathbf{E}^{(o)}$ in the incidence wave coincides with the axis z. Components E_z and E_y, H_z and H_y are not equal to zero, these values depending only upon the coordinate x.

For circular polarization

$$E_\pm = E_y \pm iE_z.$$

Maxwell equations have the form

$$\frac{\partial^2 E_\pm^{(n)}}{\partial x^2} \pm \frac{\omega^2}{c^2}\varepsilon_\pm^{(n)}E_\pm^{(n)} = 0 \quad (11)$$

where $\varepsilon_\pm^{(n)} = \varepsilon^{(n)} \pm g^{(n)}H_0$.

Using equation (1) we obtain for the transmitted waves E'_+ and E'_-

$$E'_\pm = E^{(o)}D_N^{-1}(\pm).$$

Here $D_N^{(\pm)}$ is the determinant of equation (1a), where a corresponding substitution $\varepsilon^{(n)} \to \varepsilon^{(n)} \pm g^{(n)}H_0$ is made in all Frenel coefficients $\varrho_{n\text{-}1,n}$ and phase multipliers.

After transmission

$$\mathrm{tg}\,\theta = \frac{E'_z}{E'_y} = \frac{1}{i}\frac{E'_+ - E'_-}{E'_+ + E'_-} = \frac{1}{i}\frac{D_N(-) - D_N(+)}{D_N(-) + D_N(+)}.$$

Solving this equation, we shall obtain

$$L^{-1}\theta = \frac{L^{-1}}{2i}\ln\frac{D_N(-)}{D_N(+)} = \frac{L^{-1}}{2i}\ln\frac{|D_N(-)|}{|D_N(+)|} + \frac{L^{-1}}{2}(\psi_N(-) - \psi_N(+))$$

or, using equation (1)

$$L^{-1}\theta = \frac{L^{-1}}{4i}\ln\frac{T(+)}{T(-)} + \frac{L^{-1}}{2}(\psi_N(-) - \psi_N(+)). \quad (12)$$

Here $T(+)[T(-)]$ are transmission coefficients of the left (right) circle polarized waves, correspondingly.

As is seen from equation (12), if $T(+) = T(-)$, then θ would be real; this signifies that the wave remains linearly polarized with vector $\mathbf{E}$ rotated through the angle θ to the initial direction. If $T(+) \neq T(-)$, the light has an elliptical polarization. The ratio of ellipse semi-axes is determined by relation ($b < a$)

$$\frac{b}{a} = |\mathrm{th}\,\mathrm{Im}\,\theta| = \left|\frac{T^{1/2}(+) - T^{1/2}(-)}{T^{1/2}(+) + T^{1/2}(-)}\right|, \quad (13)$$

and angle χ between the large axis of the ellipse and the axis $0y$, is

$$\chi = \mathrm{Re}\,\theta = \tfrac{1}{2}[\psi(-) - \psi(+)]. \quad (14)$$

As Thouless has shown [4], a dispersion relation exists between the length of localization and the density of states. Earlier, it was shown in [5] that this relation may be presented in the form of linear dispersion relation between $\ln|D_N|$ and the imaginary part $\mathrm{Im}\ln D_N = \psi$. That is why self-averaging of angle θ (12), i.e., the degree of wave ellipticity and angle of rotation, follows immediately from the self-averaging of localization length and density of states. Employing the connection of density states with $\mathrm{Im}\ln D_N$ [5], angle θ may be presented in the form

$$\frac{\partial}{\partial\omega}\langle\theta\rangle = \frac{1}{4i}\frac{\partial}{\partial\omega}\left\langle \ln\frac{T(+)}{T(-)}\right\rangle + \tfrac{1}{2}[\nu_+(\omega) - \nu_-(\omega)],$$

where $\nu_\pm(\omega)$ are densities of states for the left and right polarized waves. Thus, measurement of the angle of rotation from the frequency gives information on the density of states in random media.

If $r \ll 1$, we have

$$L^{-1}\theta = \frac{H\omega^2}{4c^2}\left(\frac{g_1L_1}{k_1L} + \frac{g_2L_2}{k_2L}\right) + \frac{Hn}{a_0(n+1)^2} \times \left(\frac{n-1}{n+1}\right)\left(\frac{g_2}{\varepsilon_2} - \frac{g_1}{\varepsilon_1}\right), \quad (15)$$

where L_1 and L_2 are total thicknesses of layers with dielectric constants ε_1 and ε_2 correspondingly. It is seen from equation (15) that effects of multiplied reflections inside the layers are not essential in this approximation.

It was shown in Ref. [5] that l_nD_N is the analytical function of frequency in the upper semi-plane, that is why a dispersion relation may be written for it. Hence, dispersion relation maybe likewise written for $\theta(\omega)$. Employing the dispersion relation and equation (9) for $l^{-1}(\alpha = 0)$, we shall receive for $\theta(\omega)$

$$\frac{a_0}{L}\theta(\omega) = \frac{i}{4}\left(\frac{g_2}{\varepsilon_2} - \frac{g_1}{\varepsilon_1}\right)H + \frac{g_1cH}{8a_0\varepsilon_1^{3/2}\omega} \times \psi'\left(1 + \frac{ic}{2\omega a_0\varepsilon_1^{1/2}}\right) + \frac{g_2cH}{8a_0\omega\varepsilon_2^{3/2}} \times \psi'\left(1 + \frac{ic}{2\omega a_0\varepsilon_2^{1/2}}\right). \quad (16)$$

If $k_ia_0 \gg 1$, then

$$L^{-1}\theta = \frac{iH}{4a_0}\left(\frac{g_2}{\varepsilon_2} - \frac{g_1}{\varepsilon_1}\right) + \frac{\pi^2cH}{48\omega a_0^2}\left(\frac{g_1}{\varepsilon_1^{3/2}} + \frac{g_2}{\varepsilon_2^{3/2}}\right). \quad (17)$$

Thus, from the comparison of equations (15) and (17), it follows that the effective light path in which polarization plane rotates through a given angle decreases for $(k_0a_0)^2$ times as compared to single transmission.

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