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ANOMALOUS MAGNETORESISTANCE OF SEMICONDUCTOR FILMS WITH AN ARBITRARY THICKNESS

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Quantum corrections for anomalous magnetoresistance of semiconductor film with an arbitrary thickness d are estimated. The limits of applicability of the quasi-two-dimensional case are determined.

AT PRESENT, IT BECOMES possible to study various electron relaxation times in disordered metals [1, 2], due to investigations of quantum corrections to conductivity of thin films, and especially their magnetoresistance. However, quite often criteria of quasi-two-dimensionality of the samples are badly performed. This is why it is important to understand what corrections may be given by the regard for the final thickness of samples. In the absence of a magnetic field, such a problem was solved in [3].

In the present short note we shall show that a regard for the three-dimensionality of the samples may be consecutively conducted for magnetoresistance as well, applying the method developed earlier in [4], to study the electron energy spectrum of non-homogeneous, spatially limited and layered samples. By this method, using the quasi-two-dimensional Green's function (GF) one succeeds in expressing the energy spectrum of such systems by means of GF of separate, noninteracting layers.

Quantum correction to conductivity of the three-dimensional sample is connected with "cooperon", the two-particle GF at a small summary impulse q and small transmitted energy, the the relation

$$\Delta\sigma(H, T) = -2e^2 D\pi^{-1} C_0(\mathbf{r}, \mathbf{r}; E), \qquad (1)$$

where D is the diffusion coefficient, $C_0(\mathbf{r}, \mathbf{r}; E)$ is the cooperon in a three-dimensional sample, determined by the equation

$$\left[\hbar D\left(-i\nabla - \frac{2e}{c}\mathbf{A}\right)^2 + E\right] C_0(\mathbf{r}, \mathbf{r}^1; E) = \delta(\mathbf{r} - \mathbf{r}^1).$$

Here $\mathbf{A}$ is the vector-potential of the magnetic field, $E = \hbar/\tau_\varphi$, τ_φ is the relaxation time of the wave function phase because of inelastic and spin-spin scattering.

Let us note that in the right hand side (1) $C_0(\mathbf{r}, \mathbf{r}; E)$ cooperon joins at coincident coordinates. In the spatially homogeneous upon the average system this function does not depend on coordinates. In the layer systems the situation changes, as at various layers diffusion coefficients of the electrons may differ. Besides, the cooperon at coincident coordinates may already depend upon the coordinates.

1. Let the film with thickness d be placed in the plane $x \circ y$. Let us present the correction to conductivity as $(H = 0)$ [1, 2].

$$\sigma_1(0, T) = -\frac{2e^2 D}{\pi d} \int^{1/l} \frac{q_\perp dq_\perp}{2\pi} \int_0^d C_\text{I}(z, z; E, \mathbf{q}_\perp) dz, \qquad (2)$$

where $C_\text{I}(z, z; E, \mathbf{q}_\perp)$ is the cooperon in the field expressed by means of $C_0(z, z; E, \mathbf{q}_\perp)$ and probability of reflection from the boundary [4], l is the length of free path, $\mathbf{q}_\perp$ is the two-dimensional wave vector in the plane $x \circ y$.

In the absence of magnetic field, when surface relaxation of the wave function phase is absent on the boundaries of the film, following the work [4] may be shown that

$$\int_0^d C_\text{I}(z, z; E, \mathbf{q}_\perp) dz = \frac{d}{dE} \ln \frac{\text{sh}\kappa_0 d}{C_0}, \qquad (3)$$

where

$$C_0 = \frac{1}{2\pi} \int \frac{dq_z}{E + \hbar D q_z^2 + \hbar D q_\perp^2} = \frac{1}{2\hbar D \kappa_0};$$

$$\kappa_0 = \sqrt{q_\perp^2 + L_\varphi^{-2}}, \qquad (4)$$

$$L_\varphi = \sqrt{\frac{\hbar D}{E}} \equiv \sqrt{D\tau_\varphi}.$$

Putting (3 and 4) into (2) and integrating in $q_\perp$, we shall obtain the following expression for

$$\sigma_1(0, T) = -\frac{e^2}{2\pi^2 \hbar d} \ln \frac{L_\varphi \cdot \text{sh} d/l}{l \cdot \text{sh} d/L_\varphi}.$$

coinciding with the results of the work [3].

2. Let us examine the film in a homogeneous magnetic field ($\mathbf{H} \parallel z$) and choose calibration in the form $A_y = Hx, A_x = A_z = 0$.

Correction to conductivity, by analogy with (2) may be presented

$$\sigma_1(H, T) = -\frac{2De^2}{\pi^2 d l_H^2} \sum_{n=0}^{\infty} \frac{d}{dE} \ln \frac{\operatorname{sh} \kappa_1 d}{C_1}. \quad (5)$$

Here

$$l_H = \left(\frac{\hbar C}{eH}\right)^{1/2}, \quad \hbar\Omega_c = \frac{4\hbar D}{l_H^2}, \quad x = \frac{4DeH}{\hbar c},$$

$$C_1 = \frac{1}{2\pi} \int \frac{dq_z}{E + \hbar D q_z^2 + \hbar\Omega_c(n + 1/2)} = \frac{1}{2\hbar D \kappa_1}, \quad (6)$$

$$\kappa_1 = 2 l_H^{-1} \sqrt{n + \frac{1}{x} + \frac{1}{2}}. \quad (7)$$

With regard for expressions (6) and (7), from (5) we shall obtain

$$\sigma_1(H, T) = -\frac{e^2}{2\pi^2 \hbar d} \sum_{n=0}^{\infty} \left[\frac{1}{2\left(n + \frac{1}{x} + \frac{1}{2}\right)} + \frac{d}{l_H \sqrt{n + \frac{1}{x} + \frac{1}{2}}} \operatorname{cth} \frac{2d}{l_H} \sqrt{n + \frac{1}{x} + \frac{1}{2}}\right].$$

Thus

$$\Delta\sigma(H, T) = \sigma_1(H, T) - \sigma_1(0, T) = \frac{e^2}{2\pi^2 \hbar d} \left\{\frac{1}{2} f_2(x) + \frac{d}{l_H} f_3(x) + \ln \frac{1 - e^{-2d/l}}{1 - e^{-2d/L_\varphi}} - \sum_{n=0}^{\infty} \frac{d}{l_H \sqrt{n + \frac{1}{x} + \frac{1}{2}}} \left(\operatorname{cth} \frac{2d}{l_H} \sqrt{n + \frac{1}{x} + \frac{1}{2}} - 1\right)\right\}. \quad (8)$$

Here designations [2] are used

$$f_2(x) = \ln x + \psi\left(\frac{1}{2} + \frac{1}{x}\right)$$

$$f_3(x) = \sum_{n=0}^{\infty} \left[\left\{2\left[\left(n + \frac{1}{x} + 1\right)^{1/2} - \left(n + \frac{1}{x}\right)^{1/2}\right] - \left(n + \frac{1}{x} + \frac{1}{2}\right)^{-1/2}\right\}\right].$$

$\psi(y)$ is the digamma function.

It is easy to convince that from (8) at $d \to 0$ and at $d \to \infty$ the well-known results of the work [1, 5] are received.

Expression (8) allows studying corrections to anomalous magnetoresistance of a film with an arbitrary thickness d. Besides, it allows to establish also the limits of applicability of the quasi-two-dimensional case.

Numeral valuations carried out for the limits $x \ll 1$ show that already at $d = 0, 2l_H$ corrections to quasi-two-dimensional formulas comprise 25% and at $d \approx 0, 3l_H$ more per cent.

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