1. (a) Find numbers \( d \) and \( r \) with \( 0 \leq r < 9 \) such that
\[
43,682 = 9d + r.
\]

Solution. Dividing 43,682 by 9, we obtain that the quotient is 4,853 with remainder \( r = 43,682 - 9 \times 4,853 = 5 \). Hence, with \( d = 4,853 \) and \( r = 5 \), we have
\[
43,682 = 9d + r.
\]

(b) A forester has 43,682 seedlings to be planted. Can these be planted in an equal number of rows with 9 seedlings in each row?

Solution. By the above, since the remainder equals 5, the answer is no.

2. Is 527 prime? Show your work. Yes-No answers will not count.

[Hint: \( 22^2 < 527 < 23^2 \).]

Solution. We know that if \( p \mid 527 \), then \( p^2 \leq 527 \). Thus if a prime \( p \), other than 527, divides it, then \( p \) can be only 2, 3, 5, 7, 11, 13, 17, or 19. Checking these numbers, we see that 17 \( \mid 527 \): 527 = 17 \times 31. Thus, 527 is NOT prime.

3. Use the Euclidean Algorithm to find GCD(276,800) and then LCM(276,800).

\[
\begin{align*}
800 & = 276 \cdot 2 + 248 \\
276 & = 248 \cdot 1 + 28 \\
248 & = 28 \cdot 8 + 24 \\
28 & = 24 \cdot 1 + 4 \\
24 & = 4 \cdot 6 + 0
\end{align*}
\]

Hence, GCD(276,800) = 4. Since GCD(276,800) \times LCM(276,800) = 276 \times 800 = 220,800, it follows that LCM(276,800) = 220,800 / 4 = 55,200.
4. 5.1A.9. On January 1st, Jane’s bank balance was $300. During the month, she wrote checks for $45, $55, $165, $35 and $100 and made deposits of $75, $25 and $400.

(a) If a check is represented by a negative integer and a deposit by a positive integer, express Jane’s transactions as a sum of positive and negative integers.

*Solution.* Let $T$ stand for “transactions”. Then
\[ T = -45 - 55 - 165 - 35 - 100 + 75 + 25 + 400. \]

(b) What was the balance in Jane’s account at the end of the month?

*Solution.*
\[
T = (-45 - 55 - 165 - 35 - 100) + (75 + 25 + 400)
= (-400) + (500)
= 100
\]

Thus the balance in Jane’s account at the end of the month, is $100 + $300 = $400.

5. 5.2A.10.Like-2. Write a numeric expression that describes the temperature of a certain body after $t$ minutes after 5:00 am if at 5:00 am the temperature of the body was $30^\circ$ and it decreases every minute by $2^\circ$.

*Solution.* After 1 minute, the temperature is $30 - 1 \times 2$.
After 2 minutes, the temperature is $30 - 2 \times 2$.
Therefore, after $t$ minutes, the temperature is $30 - t \times 2 = 30 - 2t$ degrees.

6. 8-2A.2.Like-2. Translate the following information into an algebraic form, and then simplify your answer: Take any number, add 5 to it, multiply the sum by 5, subtract 10, and divide by 5. Finally, subtract the original number.

*Solution.* Let $n$ be the number. Add 5 to it: $n + 5$. Multiply the sum by 5: $5(n + 5)$. Subtract 10: $5(n + 5) - 10$. Divide by 5: \( \frac{5(n+5)-10}{5} \). Subtract the original number: \( \frac{5(n+5)-10}{5} - n \). Simplify:
\[
\frac{5(n + 5) - 10}{5} - n = \frac{(5n + 25) - 10}{5} - n
\]
\[
= \frac{5n + (25 - 10)}{5} - n
\]
\[
= \frac{5n + 15}{5} - n
\]
\[
= \frac{5(n + 3)}{5} - n
\]
\[
= (n + 3) - n
\]
\[
= 3
\]

7. Explain in detail whether or not 2640 is divisible by 2, 3, 4, 5, 6, 8, 9, 10, and 11. Show your work.

\textit{Solution.}

(a) It is even. Therefore it is divisible by 2.

(b) \(2+6+4+0=12\), which is divisible by 3. Therefore it is divisible by 3.

(c) 40, the number formed by the last two digits, is divisible by 4. Therefore it is divisible by 4.

(d) 0, the last digit, is divisible by 5. Therefore it is divisible by 5.

(e) Because 2640 is divisible by 2 and 3, it is divisible by 6.

(f) 640, the number formed by the last three digits, is divisible by 8. Therefore it is divisible by 8.

(g) \(2+6+4+0=12\), which is NOT divisible by 9. Therefore it is NOT divisible by 9.

(h) Because the last digit is 0, it is divisible by 10.

(i) The sum of even power digits, 2+4=6. The sum of odd power digits, 6+0=6. Since 6-6=0 is divisible by 11, 2640 is divisible by 11.

Therefore, 2640 is divisible by 2, 3, 4, 5, 6, 8, 10, and 11, but not by 9.
8. **5.2A.22.Like.** Using the difference of squares formula, find the following:

(a) $99 \times 101$

*Solution.*

$$99 \times 101 = (100 - 1)(100 + 1) = 100^2 - 1^2 = 10000 - 1 = 9999.$$  

(b) $19 \times 21$

*Solution.*

$$19 \times 21 = (20 - 1)(20 + 1) = 20^2 - 1^2 = 400 - 1 = 399.$$  

9. The Radio Station MATH221 “La Primorosa” gave away a discount coupon for every twelfth and thirteenth caller. Every eighteenth caller received free concert tickets. Which caller was first to get both a coupon and a concert ticket?

*Solution.* The first caller to get both a coupon and a concert ticket was the smallest of the $\text{LCM}(12,18)$ or $\text{LCM}(13,18)$. Since $12 = 2^2 \times 3$, and $18 = 2 \times 3^2$, we see that $\text{LCM}(12,18) = 2^2 \times 3^2 = 36$. Since 13 and 18 are relatively prime, $\text{LCM}(13,20) = 208$. Since $36 < 208$, we conclude that the first caller to get both a coupon and a concert ticket was the 36th.

10. (a) In less than 30 words define what is the least common multiple of two numbers. No examples will count.

*Solution.* The least common multiple of two numbers is the smallest positive number that is a multiple of each of those numbers.

(b) In less than 30 words define what is the greatest common divisor of two numbers. No examples will count.

*Solution.* The greatest common divisor of two numbers is the greatest positive number that divides each of those numbers.