Math 440
HW #7

Section 10.1 & 10.2 #2, 3, 5, 6

#2 $n=20$ $Y=$ # for whom drug dose induces sleep

$H_0: p=.8$ vs. $H_a: p<.8$ $R.R=\{y\le 12\}$

a) What is type I error?

* Reject claim, $p=.8$, when claim is true.

b) Find α

$P(Y\le 12 \text{ when } p=.8)$

$Y\sim Bin(20,.8) = \sum_{i=0}^{12}\left(\frac{20!}{y!(20-y)!}\right)(.8)^y(.2)^{20-y} = \boxed{0.03214}$ ✓

c) What is type II error?

* Failed to reject claim, $p=.8$, when claim H_a is true, $p<.8$.

d) Find β when $p=.6$

$Y\sim Bin(20,.6) = \sum_{i=13}^{20}\left(\frac{20!}{y!(20-y)!}\right)(.6)^y(.4)^{20-y} = \boxed{0.4159}$ ✓

e) Find β when $p=.4$

$Y\sim Bin(20,.4) = \sum_{i=13}^{20}\left(\frac{20!}{y!(20-y)!}\right)(.4)^y(.6)^{20-y} = \boxed{0.02103}$ ✓

#3 Refer to Exercise 10.2

a) Find the R.R. of the form $\{y\le c\}$ so $\alpha\approx .01$

* when $C=11$, $\alpha=0.009982$

b) For R.R. in (a), find β when $p=.6$

$Y\sim Bin(20,.6) = \sum_{i=12}^{20}\left(\frac{20}{y!(20-y)!}\right)(.6)^y(.4)^{20-y} = \boxed{0.5956}$ ✓

c) For R.R. in (a), find β when $p=.4$

$Y\sim Bin(20,.4) = \sum_{i=12}^{20}\left(\frac{20!}{y!(20-y)!}\right)(.4)^y(.6)^{20-y} = \boxed{0.0565}$ ✓

#5 Y_1 & $Y_2 \sim Uniform(\theta,\theta+1)$ $H_0: \theta=0$ vs. $H_a: \theta>0$

Test 1: Reject H_0 if $Y_1>.95$

Test 2: Reject H_0 if $Y_1+Y_2>c$

$U=Y_1+Y_2$, $f(u)=\begin{cases} u & 0\le u\le 1 \\ 2-u & 1<u\le 2\end{cases}$

$cdf=\begin{cases} u^2 & 0\le u\le 1 \\ 2u-\frac{1}{2}u^2 & 1<u\le 2\end{cases}$

Find c so test 2 has same value for α as test 1.

Test 1: $P(Y_1>.95)=.05=\alpha$

Test 2: $\alpha=.05$, $P(U>c)=\int_c^2(2-u)\,du = \frac{1}{2}c^2-2c+2$

$\frac{1}{2}c^2-2c+2=0.05$

$\boxed{C=1.6838}$ ✓

#6 $n=36$ $Y=$ # of times coin lands on head

$H_0: p=.5$ vs. $H_a: p \neq .5$ R.R.: $|y-18| \geq 4$

a) What is value of α?

$P(|y-18| \geq 4 \text{ when } p=.5) = P(y \geq 22) + P(y \leq 14)$

$Y \sim Bin(36, .5) = \sum_{i=22}^{36} \binom{36}{y}(.5)^y(.5)^{36-y} + \sum_{i=0}^{14} \binom{36}{y}(.5)^y(.5)^{36-y} = \boxed{0.24299}$

b) What is value of β if $p=.7$?

$P(15 \leq y \leq 21)$

$Y \sim Bin(36, .7) = \sum_{i=15}^{21} \binom{36}{y}(.7)^y(.3)^{36-y} = \boxed{0.091554}$

Section 10.3 # 17, 20, 22, 27, 32, 33

#17

	Exclusively Breaststroke	Individual Medley
sample size	130	80
Sample mean (m)	9017	5853
sample s.d. (m)	7162	1961
Population mean	μ_1	μ_2

Is there sufficient evidence to indicate that the average number of meters per week spent practicing breaststroke is greater for exclusive breaststrokers than it is for those swimming individual medley?

a) state the null and alternative hypothesis

$\boxed{H_0: \mu_1 = \mu_2 \quad vs. \quad H_1: \mu_1 > \mu_2}$

b) What is the appropriate rejection region for $\alpha = 0.01$ level test?

$\boxed{\text{reject if } z > 2.33}$

c) Calculate the observed value of appropriate test statistic.

$z\text{-score} = (\bar{x}_1 - \bar{x}_2) - D_0 / \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = (9017-5853)-0 \Big/ \sqrt{\frac{7162^2}{130} + \frac{5853^2}{80}}$

$= \boxed{3.488}$

d) Conclusion?

* Reject Ho under sig. level $\alpha = 0.01$

e) Practical reason for conclusion reached?

* z-score was greater than α, so Ho must

#20 $n=50$ $\mu=64$ $\sigma=8$ $\bar{x}=62$

$H_0: \mu=64$ vs. $H_1: \mu<64$ RR$=.01$

Z-score $= \bar{x}-\mu_0/\sigma/\sqrt{n}$

$= 62-64/8/\sqrt{50}$

$= \boxed{-1.76777}$ ✓

If z-score is less than -2.325, it must be rejected.

*Failed to reject manufacturers claim, H_0, under sig. level $\alpha=0.01$

#22 $n_1=368 \Rightarrow$ Traditional method

$n_2=372 \Rightarrow$ active oriented method

a) W/o data, expect difference in mean pretest? What alternative hypothesis would you choose to test?

*Yes, $H_0: \mu=\mu_0$ vs. $H_1: \mu \neq \mu_0$

I would expect students who were taught using the active oriented method to have higher test scores.

b) Does (a) correspond to one-tailed or two-tailed stat. test?

$\boxed{\text{* two tailed}}$

c) $\bar{x}_1=14.06$ $\sigma_1=5.45$ $n_1=368$ $\alpha=.01$ two sided

$\bar{x}_2=13.38$ $\sigma_2=5.59$ $n_2=372$ $\alpha/2=.005$

$= 2.576$

Z-score $= (\bar{x}_1-\bar{x}_2)-D_0 / \sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$

$= (14.06-13.38) / \sqrt{\frac{5.45^2}{368}+\frac{5.59^2}{372}}$

$= \boxed{1.6755}$ ✓

If z-score is outside $(-2.572, 2.576)$, Ho must be rejected.

$\boxed{\text{* Fail to reject Ho under } \alpha=.01}$ ✓

#27 Do the data indicate a significant difference in the 2003 proportions of students who are fluent in English for two districts?

District	Riverside	Palm Springs
# of students tested	$n_1 = 6124$	$n_2 = 5512$
Percentage fluent	$\hat{p}_1 = 40$	$\hat{p}_2 = 37$

$\alpha = 0.01 \Rightarrow 2.33/2 = 1.165$

$H_0: p_1 - p_2 = 0$ ($\Rightarrow p_1 = p_2 = p$) vs. $p_1 - p_2 \neq 0$

$\hat{p} = \frac{(0.4)(6124) + (.37)(5512)}{6124 + 5512} = .385$

$$Z\text{-score} = (\hat{p}_1 - \hat{p}_2) - D_0 \Big/ \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$
$$= (\hat{p}_1 - \hat{p}_2) - D_0 \Big/ \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}$$
$$= (0.4 - .37) \Big/ \sqrt{\frac{(.385)(1-.385)}{6124} + \frac{0.385(1-.385)}{5512}} = \boxed{3.32}$$

* Reject H_0 under sig. level $\alpha = 0.01$

✓

#32 $n = 1060$ $\hat{p} = .54$ fair or poor rating

$H_0: p = .50$ vs. $H_1: p > .50$

$$Z\text{-score} = \hat{p} - p_0 \Big/ \sqrt{\frac{p_0(1-p_0)}{n}}$$
$$= .54 - .50 \Big/ \sqrt{\frac{.5(.5)}{1060}} = \boxed{2.6046}$$

✓

R.R.: $\alpha = .05$ $Z_\alpha = 1.645$ Reject if $Z > 1.645$

$\boxed{\therefore \text{Reject } H_0 \text{ under sig. level } \alpha = .05}$

✓

#33 $n_1 = 200$ $H_0: P_1 = P_2$ vs. $P_1 > P_2$ $\alpha = 0.05$ (1.645)

$n_2 = 200$

$\hat{p}_1 = \frac{46}{200} = .23$

$\hat{p}_2 = \frac{34}{200} = .17$

$\hat{p} = \frac{.23(200) + .17(200)}{400}$

$= .2$

$$Z\text{-score} = (\hat{p}_1 - \hat{p}_2) - D_0 \Big/ \sqrt{\frac{p_1(1-p_1)}{n} + \frac{p_2(1-p_2)}{n}}$$
$$= (\hat{p}_1 - \hat{p}_2) - D_0 \Big/ \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}$$
$$= (.23 - .17) - 0 \Big/ \sqrt{\frac{.2(.8)}{200} + \frac{.2(.8)}{200}}$$
$$= \boxed{1.5}$$

$\boxed{\text{* Fail to reject } H_0 \text{ under sig. level } \alpha = 0.05}$

✓

Section 10.4 #38, 39

#38 Refer to Exercise #20, Using R.R found, find β for specific alternative $\mu_a = 60$.

$\bar{x} - \mu_0 / \sigma/\sqrt{n} > 2.33 \Rightarrow$ Reject Ho

$\bar{x} - 62 / 8/\sqrt{50} > 2.33$

$\bar{x} > 64.0594 \Rightarrow$ Reject Ho

$R.R = \{\bar{x} > 64.0594\}$

$\beta = P(\bar{x} < 64.0594$ when $\mu = 60)$

$= P\left(\bar{x} - \mu / \sigma/\sqrt{n} < \frac{64.0594 - \mu}{\sigma/\sqrt{n}}\right)$

$= P\left(z < \frac{64.0594 - 60}{8/\sqrt{50}}\right)$

$= P(z < 3.588)$

$\boxed{\beta = .0005}$ X

Solution:
$H_0 = \mu \geq 64$
v.s $H_1 = \mu < 64$

Reject Ho if

$z = \frac{\bar{x} - 64}{\sigma/\sqrt{n}} < -2.326$

or if $\bar{y} < 64 - \frac{2.326\sigma}{\sqrt{n}} = 61.36$

So, $\beta = P(\bar{x} > 61.36 \mid \mu = 60) = P\left(z > \frac{61.36 - 60}{8/\sqrt{50}}\right)$

$= P(z > 1.2) = 0.115$

#39 Refer to Exercise #30. Calculate β for alternative $p_a = .15$

$n = 100 \quad P_0 = .2 \quad \alpha = 0.05$

$\hat{p} - p_0 / \sqrt{\frac{p_0(1-p_0)}{n}} < \alpha$

$\hat{p} - .2 / \sqrt{\frac{.2(.8)}{100}} < 1.645 \qquad \hat{p} < 0.2658$

$\beta = P(\hat{p} > 0.2658$ when $p_a = .15)$

$= P\left(z \leq \frac{.2658 - .15}{\sqrt{(.15)(.85)/100}}\right)$

$= P(z \leq 3.2066)$

$\boxed{\beta = 0.000672}$ X

Solution: in Ex. 10.30, we found $RR = \{\hat{p} < 0.1342\}$

$\beta = P(\hat{p} > 0.1342 \mid p = 0.15) = P\left(z > \frac{0.1342 - 0.15}{\sqrt{0.15(0.85)/100}}\right)$

$= P(z > -0.4424) = 0.67$

10.50 Let μ = mean occupancy rate. To test H_0: $\mu \geq .6$, H_a: $\mu < .6$, the computed test statistic is $z = \frac{.58-.6}{.11/\sqrt{120}} = -1.99$. The p–value is given by $P(Z < -1.99) = .0233$. Since this is less than the significance level of .10, H_0 is rejected.

10.51 To test H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 \neq 0$, where μ_1, μ_2 represent the two mean reading test scores for the two methods, the computed test statistic is

$$z = \frac{74-71}{\sqrt{\frac{9^2}{50}+\frac{10^2}{50}}} = 1.58.$$

The p–value is given by $P(|Z| > 1.58) = 2P(Z > 1.58) = .1142$, and since this is larger than $\alpha = .05$, we fail to reject H_0.

10.52 The null and alternative hypotheses are H_0: $p_1 - p_2 = 0$ vs. H_a: $p_1 - p_2 > 0$, where p_1 and p_2 correspond to normal cell rates for cells treated with .6 and .7 (respectively) concentrations of actinomycin D.

a. Using the sample proportions .786 and .329, the test statistic is (refer to Ex. 10.27) $z = \frac{.786-.329}{\sqrt{(.557)(.443)\frac{2}{70}}} = 5.443$. The p–value is $P(Z > 5.443) \approx 0$.

b. Since the p–value is less than .05, we can reject H_0 and conclude that the normal cell rate is lower for cells exposed to the higher actinomycin D concentration.

10.54 **a.** The hypotheses are H_0: $p = .85$, H_a: $p > .85$, where p = proportion of right–handed executives of large corporations. The computed test statistic is $z = 5.34$, and with $\alpha = .01$, $z_{.01} = 2.326$. So, we reject H_0 and conclude that the proportion of right–handed executives at large corporations is greater than 85%

b. Since p–value = $P(Z > 5.34) < .000001$, we can safely reject H_0 for any significance level of .000001 or more. This represents strong evidence against H_0.

10.69 The hypotheses are H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 \neq 0$.

a. The computed test statistic is, where $s_p^2 = \frac{10(52)+13(71)}{23} = 62.74$, is given by

$$t = \frac{64-69}{\sqrt{62.74\left(\frac{1}{11}+\frac{1}{14}\right)}} = -1.57.$$

i. With $11 + 14 - 2 = 23$ degrees of freedom, $-t_{.10} = -1.319$ and $-t_{.05} = -1.714$. Thus, since we have a two–sided alternative, $.10 < p\text{–value} < .20$.

ii. Using the Applet, $2P(T < -1.57) = 2(.06504) = .13008$.

b. We assumed that the two samples were selected independently from normal populations with common variance.

c. Fail to reject H_0.

10.76 **a.** To test H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 > 0$, where μ_1, μ_2 represent mean plaque measurements for the control and antiplaque groups, respectively.

b. The pooled sample variance is $s_p^2 = \frac{6(.32)^2 + 6(.32)^2}{12} = .1024$ and the computed test statistic is $t = \frac{1.26 - .78}{\sqrt{.1024\left(\frac{2}{7}\right)}} = 2.806$ with 12 degrees of freedom. Since $\alpha = .05$, $t_{.05} = 1.782$ and H_0 is rejected: there is evidence that the antiplaque rinse reduces the mean plaque measurement.

c. With $t_{.01} = 2.681$ and $t_{.005} = 3.005$, $.005 < p$–value $< .01$ (exact: .00793).