Math 4200 Student Activity 3

Properties of Estimators and the Methods of Estimation

1. Let $Y_1, \ldots, Y_n$ be a random sample from a $Unif(0, \theta)$. Let

$$\hat{\theta}_1 = \frac{n+1}{n}(Y_{(n)}) \qquad \hat{\theta}_2 = 3\bar{Y}$$

   (a) Is $\hat{\theta}_1$ an unbiased estimator for θ? Justify your answer.

   (b) Is $\hat{\theta}_2$ an unbiased estimator for θ? Justify your answer.

   (c) Find the efficiency of $\hat{\theta}_2$ relative to $\hat{\theta}_1$.

2. Let $Y_1, \ldots, Y_n$ be a random sample from the probability density function

$$f(y|\theta) = \theta y^{\theta-1}, \qquad 0 \leq y \leq 1$$

   where $\theta > 0$. Show that $\overline{y}$ is a consistent estimator of $\frac{\theta}{\theta+1}$.

3. Let $Y_1, \ldots, Y_n$ be a random sample from a population with the following pdf:

$$f(y|\alpha, \beta) = \alpha\beta^{\alpha}y^{-(\alpha-1)}, \quad y \geq \beta$$

   This distribution is known as the Pareto distribution.

   (a) Find the likelihood function.

   (b) If α is known, show that $Y_{(1)}$ is sufficient for β.

   (c) Show that $\prod_{i=1}^{n} Y_i$ and $Y_{(1)}$ are jointly sufficient for α and β.

4. Let $Y_1, \ldots, Y_n$ be a random sample from a normal distribution with mean μ and variance 1.

   (a) Show that the MVUE of μ^2 is $\widehat{\mu^2} = \bar{y}^2 - 1/n$.

5. Let $Y_1, \ldots, Y_n$ be a random sample from the population with pdf

$$f(y|\theta) = \frac{1}{(r-1)!\theta^r}e^{-y/\theta}y^{r-1}, \qquad y > 0$$

   (a) Find an MVUE for θ.

6. Let $Y_1, \ldots, Y_n$ be a random sample from a $Pois(\lambda = \theta)$.

   (a) Determine the MLE of θ.

   (b) Determine the MLE of the standard deviation of the distribution.

7. Let $Y_1, \ldots, Y_n$ be a random sample from the population with pdf

$$f(y|\theta) = \frac{1}{\theta} e^{-y/\theta}, \qquad y > 0$$

   (a) Find the MOM estimator for θ.

   (b) Find the MLE of θ.

   (c) Find the MLE of $P(Y \leq 2)$.

   (d) Find the MLE of the median of the distribution.