

Math 4200 Student Activity #1

Transformation, Sampling Distribution and CLT

1. If X has an exponential distribution with mean θ , show that $V = \frac{2X}{\theta}$ has a chi-square distribution with 2 degrees of freedom. (Hint: Use MGF)

2. Let X_1, X_2, \dots, X_n be independent uniform random variable such that $X_i \stackrel{iid}{\sim} Unif(0, \theta)$. Find the pdf of $U = \frac{X_{(n)}}{\theta}$, where $X_{(n)}$ is the order statistic.

3. A plant supervisor is interested in budging weekly repair costs for a certain type of machine. Records over the past years indicate that these repair costs have an exponential distribution with mean 20 for each machine studied. Let X_1, X_2, \dots, X_5 denote the repair costs for give of these machines for the next week.
 - (a) Find the pdf for $U = \sum_{i=1}^5 X_i$
 - (b) Find $E(\sum_{i=1}^5 X_i)$ and $Var(\sum_{i=1}^5 X_i)$
 - (c) Find $E(\bar{X})$ and $Var(\bar{X})$
 - (d) Find a number c such that $P(\sum_{i=1}^5 X_i > c) = 0.05$. (Hint: use the result from problem #1)

4. Let X and Y be independent random variables. Suppose that $X \sim N(5, 15^2)$ and $Y \sim \chi^2(5)$. Determine $P(X - 5 > 2\sqrt{Y})$

5. From each of two normal populations with identical means and with standard deviations of 6.4 and 7.2, independent random samples of 64 observations are drawn. Find the probability that the difference between the means of the samples exceeds 0.6 in absolute value.

6. Let X_1, X_2, \dots, X_{10} denote a random sample of size 10 from a normal distribution with mean 0 and variance σ^2 . Use the following steps to find the number c such that $P(-c < \frac{s}{\bar{x}} < c) = 0.95$, where s is the sample std deviation and \bar{x} is the sample mean.

- (a) Find the distribution of $\frac{10\bar{x}}{s^2}$
- (b) Find the distribution of $\frac{s^2}{10\bar{x}}$
- (c) Find the number c based on (b)

7. Shear strength measurements for spot welds have been found to have standard deviation 10 pounds per square inch (psi). If 100 test welds are to be measured, what is the approximate probability that the sample mean will be within 1 psi of the true population mean?
8. Suppose that X_1, X_2, \dots, X_{40} denote a random sample of measurements on the proportion of impurities in iron ore samples that are owned by a local mining company. Let each variable X_i have the following pdf

$$f(x) = 3x^2, \text{ for } 0 \leq x \leq 1.$$

The ore is to be rejected by the potential buyer if \bar{x} exceeds 0.7. Find $P(\bar{x} > 0.7)$ for a sample size of 40. Do the results suggest that the buyer would make a purchase?

9. Let $X_i \sim iid N(i, i^2)$, for $i = 1, 2, 3$. For each of the following situations, use the X_i 's to construct a statistic with the indicated distribution.
- Chi squared with 3 degrees of freedom
 - T distribution with 2 degrees of freedom
 - F distribution with 1 and 2 degrees of freedom
10. If \bar{X}_1 and \bar{X}_2 are the means of two independent samples of size n from a population with variance σ^2 , find a value for n so that $P(|\bar{X}_1 - \bar{X}_2| < \frac{\sigma}{5}) \approx 0.99$. Justify your calculations.