1. [6 *pts*] A public health official is planning for the supply of influenza vaccine needed for the upcoming flu season. She took a poll of 350 local citizens and found that only 126 said they would be vaccinated. Construct a 98% confidence interval for the true proportion of people who plan to get the vaccine.

2. [10 *pts*] Someone claims that X-rays may penetrate the tooth enamel of men and women differently, a fact that allows dental structure to help identify the sex of badly decomposed bodies. Listed in the table below are the enamel spectropenetration gradients for eight male teeth and eight female teeth. These numbers are measures of the rate of change in the amount of X-ray penetration through a 500-micron section of tooth enamel at a wavelength of 600nm as opposed to 400nm. The sample mean is 5.4 for the males, and 4.5 for the females. The sample variance for males and females are 0.55 and 0.58, respectively.

Male	4.9	5.4	5.0	5.5	5.4	6.6	6.3	4.3
Female	4.8	5.3	3.7	4.1	5.6	4.0	3.6	5.0

   (a) [6 *pts*] Construct a 95% confidence interval for the difference in the mean enamel spectropenetration gradients for the two genders.

   (b) [4 *pts*] Set up a 99% upper confidence bound for the standard deviation of the mean enamel spectropenetration gradients for the male group.

3. [15 *pts*] Let $X_1, \ldots, X_n$ denote a random sample from the exponential distribution with the probability density function

$$f(x|\lambda) = \lambda e^{-\lambda x}, \quad x > 0.$$

   Let $\hat{\lambda}_1 = \bar{X}$ and $\hat{\lambda}_2 = nX_{(1)}$, where $X_{(1)} = min(X_1, \ldots, X_n)$

   (a) [5 *pts*] Is $\hat{\lambda}_1$ a consistent estimator for $1/\lambda$?

   (b) [6 *pts*] Does $\hat{\lambda}_2$ convergence in probability to $1/\lambda$?

   (c) [4 *pts*] Find the relative efficiency of $\hat{\lambda}_1$ to $\hat{\lambda}_2$. Which one is relatively more efficient? Why?

4. [16 *pts*] Let $X_1, \ldots, X_n$ be a random sample from the probability density function

$$f(x|\mu, \sigma) = \frac{1}{\sigma} e^{-(x-\mu)/\sigma}, \quad \mu < x < \infty, \quad 0 < \sigma < \infty$$

   (a) [6 *pts*] Find the joint sufficient statistics for μ and σ.

   (b) [6 *pts*] Find the MLE of μ and σ, respectively. Use the graph to support your answer, if needed.

   (c) [4 *pts*] Is MLE always some functions of sufficient statistics? Prove or disprove it.

5. [6 *pts*] Let $X_1, \ldots, X_n$ be a random sample from the **exponential family** with the probability density function given by

$$f(x|\theta) = h(x)c(\theta)e^{w(\theta)t(x)}$$

where h and t are functions only about X, and c and w are functions that only about θ. Show that $\sum_{i=1}^{n} t(x_i)$ is sufficient for θ.

6. [15 *pts*] Let $X_1, \ldots, X_n$ be a random sample from the probability density function given by

$$f(x|\theta) = \frac{\theta}{(1+x)^{1+\theta}}, \quad 0 < x < \infty, \quad \theta > 0$$

   (a) [5 *pts*] Show that this probability density function belongs to the exponential family which has the pdf defined in problem 5 by identifying the h, c, w, and t functions.

   (b) [5 *pts*] Find a sufficient statistics for θ.

   (c) [5 *pts*] Find the Maximum Likelihood Estimation (MLE) of θ.

7. [16 *pts*] Let $X_1, \ldots, X_n$ be a random sample from a Gamma distribution with the unknown parameter β. The probability density function is given by

$$f(x|\beta) = \frac{x^3 e^{-x/\beta}}{6\beta^4}, \qquad x > 0$$

   (a) [6 *pts*] Find the Maximum Likelihood Estimation (MLE) of β.

   (b) [5 *pts*] Find the MVUE for β.

   (c) [5 *pts*] Find the MVUE for β^2.

8. [16 *pts*] Let $X_1, \ldots, X_n$ be a random sample from a uniform $(\theta, 2\theta)$ distribution with the probability density function

$$f(x|\theta) = \frac{1}{\theta}, \quad 0 < \theta \le x \le 2\theta$$

   (a) [5 *pts*] Find the Method of Moments (MOM) estimator of θ.

   (b) [6 *pts*] Find the Maximum Likelihood Estimation (MLE) of θ, use a graph to support your answer if needed.

   (c) [5 *pts*] Find the Maximum Likelihood Estimation (MLE) of the median of the distribution.