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Charged Two-Dimensional Magnetoexciton and Two-Mode Squeezed Vacuum States[1]

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A novel unitary transformation of the Hamiltonian that allows one to partially separate the center-of-mass motion for charged electron–hole systems in a magnetic field is presented. The two-mode squeezed oscillator states that appear at the intermediate stage of the transformation are used for constructing a trial wave function of a two-dimensional charged magnetoexciton. *© 2001 MAIK "Nauka/Interperiodica".*

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A problem of center-of-mass (CM) separation for a quantum-mechanical system of charged interacting particles in a magnetic field B was studied by many authors [1–5]. When a charge-to-mass ratio is the same for all particles, the CM and internal motions decouple [2–4] in B. For a neutral system, the CM coordinates can be separated [1, 2] in the Schrödinger equation. This is associated with the fact that translations commute for a neutral system in B. In general, only a partial separation of the CM in magnetic fields is possible [2, 4, 5]. In this letter, we propose a novel operator approach for performing such a separation in charged electron–hole (e–h) systems in B. This approach can be useful for studying in strong magnetic fields, e.g., atomic ions with not-too-large mass ratios [4] and charged excitations in two-dimensional (2D) electron systems, in particular, in the fractional quantum Hall effect regime in planar geometry [6, 7]. In this work, we study in 2D a three-particle problem of two electrons and one hole in a strong magnetic field, i.e., a negatively charged magnetoexciton X^- (see [5, 8, 9] and references therein). We consider an approximate X^- ground state in the form which is related to the two-mode squeezed [10] oscillator vacuum states.

The Hamiltonian describing the 2D three-particle $2e$–h complex in a perpendicular magnetic field B is $H = H_0 + H_{\rm int}$, where the free-particle part is given by

$$H_0 = \sum_{i=1,2}\frac{\hat{\mathbf{\Pi}}_{ei}^2}{2m_e} + \frac{\hat{\mathbf{\Pi}}_h^2}{2m_h} \equiv \sum_{i=1,2} H_{0e}(\mathbf{r}_i) + H_{0h}(\mathbf{r}_h), \quad (1)$$

and $\hat{\mathbf{\Pi}}_j = -i\hbar\nabla_j - \frac{e_j}{c}\mathbf{A}(\mathbf{r}_j)$ are kinematic momentum operators. The interaction Hamiltonian $H_{\rm int} = H_{ee} + H_{eh}$ is

$$H_{ee} = \frac{e^2}{\epsilon|\mathbf{r}_1 - \mathbf{r}_2|}, \quad H_{eh} = -\sum_{i=1,2}\frac{e^2}{\epsilon|\mathbf{r}_i - \mathbf{r}_h|}. \quad (2)$$

The Hamiltonian H commutes [2, 4, 9] with the operator of magnetic translations (MT) $\hat{\mathbf{K}} = \sum_j \hat{\mathbf{K}}_j$, where $\hat{\mathbf{K}}_j = \hat{\mathbf{\Pi}}_j - \frac{e_j}{c}\mathbf{r}_j \times \mathbf{B}$. In the symmetric gauge, $\mathbf{A} = \frac{1}{2}\mathbf{B}\times\mathbf{r}$, the operators satisfy the relation $\hat{\mathbf{K}}_j(\mathbf{B}) = \hat{\mathbf{\Pi}}_j(-\mathbf{B})$; independent of the gauge, $\hat{\mathbf{K}}_j$ and $\hat{\mathbf{\Pi}}_j$ commute. The important feature of $\hat{\mathbf{K}}$ and $\hat{\mathbf{\Pi}} = \sum_j \hat{\mathbf{\Pi}}_j$ is the noncommutativity of the components in B: $[\hat{K}_x, \hat{K}_y] = -[\hat{\Pi}_x, \hat{\Pi}_y] = -i\frac{\hbar B}{c}Q$, where $Q = \sum_j e_j$ is the total charge. This allows one to introduce the raising and the lowering Bose ladder operators for the whole system [2, 4, 9]

$$\hat{k}_\pm = \pm\frac{i}{\sqrt{2}}(\hat{k}_x \pm i\hat{k}_y), \quad [\hat{k}_+, \hat{k}_-] = -\frac{Q}{|Q|}, \quad (3)$$

$$\hat{\pi}_\pm = \mp\frac{i}{\sqrt{2}}(\hat{\pi}_x \pm i\hat{\pi}_y), \quad [\hat{\pi}_+, \hat{\pi}_-] = \frac{Q}{|Q|}, \quad (4)$$

where $\hat{\mathbf{k}} = \sqrt{c/\hbar B|Q|}\,\hat{\mathbf{K}}$, $\hat{\boldsymbol{\pi}} = \sqrt{c/\hbar B|Q|}\,\hat{\mathbf{\Pi}}$, and the phases of the operators (3) and (4) can be chosen arbitrary. The operator $\hat{\mathbf{k}}^2$ has the discrete oscillator eigenvalues $2k + 1$, $k = 0, 1, \ldots$, which are associated [2, 4]

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with the guiding center of a charged complex in B. The values of k can be used,[3] together with the total angular momentum projection M_z and the electron, S_e, and hole, S_h, spin quantum numbers, for the classification of states [9]; the exact eigenenergies are degenerate [2, 4] in k.

In terms of the *single-particle* Bose ladder intra-Landau level (LL) operators [5, 6] $B_e^\dagger(\mathbf{r}_j) = -i\sqrt{c/2\hbar Be}(\hat{K}_{jx} - i\hat{K}_{jy})$ for the electrons and $B_h^\dagger(\mathbf{r}_h) = -i\sqrt{c/2\hbar Be}(\hat{K}_{hx} + i\hat{K}_{hy})$ for the hole, the raising operator takes the form $\hat{k}_- = B_e^\dagger(\mathbf{r}_1) + B_e^\dagger(\mathbf{r}_2) - B_h(\mathbf{r}_h)$. One needs to diagonalize $\hat{k}_-$ in order to maintain the exact MT symmetry. This can be achieved by performing first an orthogonal transformation [3, 4] of the electron coordinates $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_h\} \longrightarrow \{\mathbf{r}, \mathbf{R}, \mathbf{r}_h\}$, where $\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$ and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/\sqrt{2}$ are the electron relative and CM coordinates. In these coordinates, $\hat{k}_- = \sqrt{2}B_e^\dagger(\mathbf{R}) - B_h(\mathbf{r}_h)$ and can be considered to be a new Bose ladder operator generated by the Bogoliubov transformation [5]

$$\tilde{B}_e^\dagger(\mathbf{R}) = uB_e^\dagger(\mathbf{R}) - vB_h(\mathbf{r}_h) = \tilde{S}B_e^\dagger(\mathbf{R})\tilde{S}^\dagger, \qquad (5)$$

where the unitary operator [5, 6, 10] $\tilde{S} = \exp(\Theta\tilde{\mathcal{L}})$ and the generator $\tilde{\mathcal{L}} = B_h^\dagger(\mathbf{r}_h)B_e^\dagger(\mathbf{R}) - B_e(\mathbf{R})B_h(\mathbf{r})$. Here, Θ is the transformation angle and $u = \cosh\Theta = \sqrt{2}$, $v = \sinh\Theta = 1$. Now we have $\hat{k}_- = \tilde{B}_e^\dagger$ and $\hat{\mathbf{k}}^2 = 2\tilde{B}_e^\dagger\tilde{B}_e + 1$. The second linearly independent creation operator is

$$\tilde{B}_h^\dagger(\mathbf{r}_h) = \tilde{S}B_h^\dagger(\mathbf{r}_h)\tilde{S}^\dagger = uB_h^\dagger(\mathbf{r}_h) - vB_e(\mathbf{R}). \qquad (6)$$

A complete orthonormal basis compatible with both axial and translational symmetries can be constructed [5] as

$$\frac{A_e^\dagger(\mathbf{r})^{n_r}A_e^\dagger(\mathbf{R})^{n_R}A_h^\dagger(\mathbf{r}_h)^{n_h}\tilde{B}_e^\dagger(\mathbf{R})^k\tilde{B}_h^\dagger(\mathbf{r}_h)^l B_e^\dagger(\mathbf{r})^m|\tilde{0}\rangle}{(n_r!n_R!n_h!k!l!m!)^{1/2}} \equiv |n_r n_R n_h;\, \widetilde{klm}\rangle. \qquad (7)$$

Here, the inter-LL Bose ladder operators are given by $A_e^\dagger(\mathbf{r}_j) = -i\sqrt{c/2\hbar Be}(\hat{\Pi}_{jx} + i\hat{\Pi}_{jy})$ and $A_h^\dagger(\mathbf{r}_h) = -i\sqrt{c/2\hbar Be}(\hat{\Pi}_{hx} - i\hat{\Pi}_{hy})$; the explicit form is given in, e.g., [5, 6]. The tilde sign shows that the transformed vacuum state $|\tilde{0}\rangle$ (see below) and the transformed operators (5) and (6) are involved. In Eq. (7), the oscillator quantum number is fixed and equals k, while $M_z = n_r + n_R - n_h - k + l - m$. The permutational symmetry requires that $n_r - m$ should be even (odd) for electron spin singlet $S_e = 0$ (triplet $S_e = 1$) states; see [5] for more detail.

The transformation introduces a new vacuum state $|\tilde{0}\rangle = \tilde{S}|0\rangle$, for which, using the normal-ordered form [5, 6, 10] of $\tilde{S}$, one obtains

$$|\tilde{0}\rangle = \tilde{S}|0\rangle = \frac{1}{\cosh\Theta}\exp[\tanh\Theta B_h^\dagger(\mathbf{r}_h)B_e^\dagger(\mathbf{R})]|0\rangle. \qquad (8)$$

The coordinate representation has the form

$$\langle\mathbf{r}\mathbf{R}\mathbf{r}_h|\tilde{0}\rangle = \frac{1}{\sqrt{2}(2\pi l_B^2)^{3/2}} \times \exp\left(-\frac{\mathbf{r}^2 + \mathbf{R}^2 + \mathbf{r}_h^2 - \sqrt{2}Z^*z_h}{4l_B^2}\right), \qquad (9)$$

where $l_B = (\hbar c/eB)^{1/2}$ is the magnetic length, $Z^* = X - iY$, and $z_h = x_h + iy_h$. Equation (9) shows that $|\tilde{0}\rangle$ contains a *coherent superposition* of an infinite number of e and h states in zero LLs. In the terminology of quantum optics [10], $|\tilde{0}\rangle$ is a *two-mode squeezed state*; for particles in a magnetic field, the squeezing has a direct geometrical meaning (for studies on single-particle coherent and squeezed states in magnetic fields, see [11] and references therein). Indeed, the probability distribution function takes the factored form

$$\left|\langle\mathbf{r}\mathbf{R}\mathbf{r}_h|\tilde{0}\rangle\right|^2 = \frac{1}{2\pi l_B^2}\exp\left(-\frac{\mathbf{r}^2}{2l_B^2}\right) \times \frac{2+\sqrt{2}}{4\pi l_B^2}\exp\left[-\frac{2+\sqrt{2}}{8l_B^2}(\mathbf{R}-\mathbf{r}_h)^2\right] \times \frac{2-\sqrt{2}}{4\pi l_B^2}\exp\left[-\frac{2-\sqrt{2}}{8l_B^2}(\mathbf{R}+\mathbf{r}_h)^2\right]. \qquad (10)$$

This shows that the distribution for the relative coordinate $\mathbf{R} - \mathbf{r}_h$ is squeezed *at the expense* of that for the coordinate $\mathbf{R} + \mathbf{r}_h$, and the variances are $\langle\tilde{0}|(\mathbf{R} \pm \mathbf{r}_h)^2|\tilde{0}\rangle = 4(2 \pm \sqrt{2})l_B^2$. The squeezing enhances the e–h attraction, which will be used below for constructing a trial wave function of the 2D magnetoexciton X^-.

Let us now perform the second unitary transformation corresponding to the diagonalization of the operator $\hat{\pi}_+ = A_e^\dagger(\mathbf{r}_1) + A_e^\dagger(\mathbf{r}_2) - A_h(\mathbf{r}_h)$. This introduces a

[3] Note that the operators $\hat{\pi}_\pm$ do not commute and, in general, do not form a simple algebra with the Hamiltonian. A special case is when the charge-to-mass ratio e_j/m_j = const, and $[H, \hat{\pi}_\pm] = \mp\hbar(e_jB/m_jc)\hat{\pi}_\pm$, which corresponds to the CM separation.

new state $|\bar{0}\rangle = \bar{S}\,\tilde{S}|0\rangle = \bar{S}\,|\tilde{0}\rangle$, which corresponds to the simultaneous diagonalization of the operators $\hat{k}_-$ and $\hat{\pi}_+$; the unitary operator $\bar{S} = \exp(\Theta\overline{\mathcal{L}})$, where the generator $\overline{\mathcal{L}} = A_h^\dagger(\mathbf{r}_h)A_e^\dagger(\mathbf{R}) - A_e(\mathbf{R})A_h(\mathbf{r}_h)$. The transformations effectively introduce new coordinates, $\{\mathbf{r}, \mathbf{R}, \mathbf{r}_h\} \longrightarrow \{\mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2\}$, where $\boldsymbol{\rho}_1 = \sqrt{2}\,\mathbf{R} - \mathbf{r}_h$ and $\boldsymbol{\rho}_2 = \sqrt{2}\,\mathbf{r}_h - \mathbf{R}$, which can be presented in the matrix form

$$\begin{pmatrix} \boldsymbol{\rho}_1 \\ \boldsymbol{\rho}_2 \end{pmatrix} = \hat{F}\begin{pmatrix} \mathbf{R} \\ \mathbf{r}_h \end{pmatrix}, \quad \hat{F} = \begin{pmatrix} \cosh\Theta & -\sinh\Theta \\ -\sinh\Theta & \cosh\Theta \end{pmatrix}, \tag{11}$$

with $\cosh\Theta = \sqrt{2}$, $\sinh\Theta = 1$; the matrix $\hat{F}$ corresponds to the $SU(1, 1)$ symmetry [10]. Indeed, the inter-LL ladder operators are changed under the Bogoliubov transformations as

$$\bar{S}\begin{pmatrix} A_e^\dagger(\mathbf{R}) \\ A_h(\mathbf{r}_h) \end{pmatrix}\bar{S}^\dagger = \hat{F}\begin{pmatrix} A_e^\dagger(\mathbf{R}) \\ A_h(\mathbf{r}_h) \end{pmatrix} = \begin{pmatrix} A_e^\dagger(\boldsymbol{\rho}_1) \\ A_h(\boldsymbol{\rho}_2) \end{pmatrix}. \tag{12}$$

The intra-LL operators (5) and (6) transform according to the same representation. The coordinate representation

$$\langle \mathbf{r}\boldsymbol{\rho}_1\boldsymbol{\rho}_2|\bar{0}\rangle = \frac{1}{(2\pi l_B^2)^{3/2}}\exp\left(-\frac{\mathbf{r}^2 + \boldsymbol{\rho}_1^2 + \boldsymbol{\rho}_2^2}{4l_B^2}\right) \tag{13}$$

shows that $|\bar{0}\rangle$ is a *true vacuum* for both the intra-LL $B_e^\dagger(\boldsymbol{\rho}_1)$, $B_h^\dagger(\boldsymbol{\rho}_2)$ and the inter-LL $A_h^\dagger(\boldsymbol{\rho}_2)$, $A_e^\dagger(\boldsymbol{\rho}_1)$ operators. Now, we can perform the change of the variables $\{\mathbf{r}, \mathbf{R}, \mathbf{r}_h\} \longrightarrow \{\mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2\}$ in the basis states:

$$\begin{aligned} &|n_r n_R n_h;\, \widetilde{klm}\rangle \\ &= \frac{\bar{S}^\dagger A_e^\dagger(\mathbf{r})^{n_r} A_e^\dagger(\boldsymbol{\rho}_1)^{n_R} A_h^\dagger(\boldsymbol{\rho}_2)^{n_h} B_e^\dagger(\boldsymbol{\rho}_1)^k B_h^\dagger(\boldsymbol{\rho}_2)^l B_e^\dagger(\mathbf{r})^m|\bar{0}\rangle}{(n_r!n_R!n_h!k!l!m!)^{1/2}} \\ &\equiv \bar{S}^\dagger|\overline{n_r n_R n_h;\, klm}\rangle. \end{aligned} \tag{14}$$

The overline shows that a state is generated in the usual way by the intra- and inter-LL Bose ladder operators acting on the true vacuum $|\bar{0}\rangle$, all in the representation of the coordinates $\{\mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2\}$. The Hamiltonian H is block-diagonal in the quantum numbers k, M_z (and S_e, S_h). Due to the Landau degeneracy [2, 9] in k, it is sufficient to consider the states with $k = 0$. This effectively removes one degree of freedom and corresponds to a partial separation of the CM motion. From now on, we will consider the $k = 0$ states only, designating such states in Eq. (14) as $|\overline{n_r n_R n_h;\, lm}\rangle$. For the Hamiltonian, we arrive therefore at the unitary transformation

$$\begin{aligned} &\langle \widetilde{m_2 l_2};\, n_{h2}n_{R2}n_{r2}|H|n_{r1}n_{R1}n_{h1};\, \widetilde{l_1 m_1}\rangle \\ &= \langle \overline{m_2 l_2;\, n_{h2}n_{R2}n_{r2}}|\bar{S}H\bar{S}^\dagger|\overline{n_{r1}n_{R1}n_{h1};\, l_1 m_1}\rangle, \end{aligned} \tag{15}$$

which is the main formal result of this work.

The Coulomb interparticle interactions (2) in the coordinates $\{\mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2\}$ take the form

$$H_{ee} = \frac{e^2}{\sqrt{2}\epsilon r}, \quad H_{eh} = -\frac{\sqrt{2}e^2}{\epsilon|\boldsymbol{\rho}_2 - \mathbf{r}|} - \frac{\sqrt{2}e^2}{\epsilon|\boldsymbol{\rho}_2 + \mathbf{r}|}, \tag{16}$$

and $H_{\rm int}$ *does not depend* on $\boldsymbol{\rho}_1$. From Eq. (12), it follows that the free Hamiltonians transform as $\bar{S}H_{0e}(\mathbf{r})\bar{S}^\dagger = H_{0e}(\mathbf{r})$, $\bar{S}H_{0e}(\mathbf{R})\bar{S}^\dagger = H_{0e}(\boldsymbol{\rho}_1)$, and $\bar{S}H_{0h}(\mathbf{r}_h)\bar{S}^\dagger = H_{0h}(\boldsymbol{\rho}_2)$ and describe *new effective particles*—free e and h in a magnetic field—with the *modified interactions* (16). The Hamiltonian of the e–e interactions $H_{ee}(\sqrt{2}|\mathbf{r}|)$ does not depend on $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$ and, therefore, is invariant: $\bar{S}H_{ee}\bar{S}^\dagger = H_{ee}$. Thus, the matrix elements of the e–e interaction are easily obtained from Eq. (15): they reduce to the matrix elements $V_{n_1 m_1}^{n_1' m_1'}$ describing the interaction of the electron with a fixed negative charge $-e$:

$$\begin{aligned} &\langle \widetilde{m_2 l_2};\, n_{h2}n_{R2}n_{r2}|H_{\rm ee}|n_{r1}n_{R1}n_{h1};\, \widetilde{l_1 m_1}\rangle \\ &= \langle \overline{m_2 l_2;\, n_{h2}n_{R2}n_{r2}}|H_{\rm ee}|\overline{n_{r1}n_{R1}n_{h1};\, l_1 m_1}\rangle \\ &= \delta_{n_{R1},\, n_{R2}}\delta_{n_{h1},\, n_{h2}}\delta_{l_1,\, l_2}\delta_{n_{r1} - m_1,\, n_{r2} - m_2}\frac{1}{\sqrt{2}}V_{n_{r1}m_1}^{n_{r2}m_2}. \end{aligned} \tag{17}$$

In, e.g., zero [5] LL, $V_{0m}^{0m} = [(2m - 1)!!/2^m m!]E_0$, where $E_0 = \sqrt{\pi/2}\,(e^2/\epsilon l_B)$. The generator $\overline{\mathcal{L}}$ and the Hamiltonian $H_{eh}(\mathbf{r}, \boldsymbol{\rho}_2)$ do not form a closed algebra of a finite order. Therefore, the explicit form of $\bar{S}H_{eh}\bar{S}^\dagger$ cannot be found. We can find, however, the form of the matrix elements of $\bar{S}H_{eh}\bar{S}^\dagger$ in Eq. (15). Because of the electron permutational symmetry $\mathbf{r} \longleftrightarrow -\mathbf{r}$, it is sufficient to consider the term $U_{eh}(\boldsymbol{\rho}_2 - \mathbf{r}) = -e^2/\epsilon|\boldsymbol{\rho}_2 - \mathbf{r}|$. Here, we only consider the states in zero LL $|\overline{000;\, lm}\rangle \equiv |\overline{lm}\rangle$. Using the normal-ordered form of $\bar{S}$, we have

$$\begin{aligned} &\langle \overline{m_2 l_2}|\bar{S}U_{eh}\bar{S}^\dagger|\overline{l_1 m_1}\rangle \equiv \bar{U}_{0m_1 0l_1}^{0m_2 0l_2} \\ &= \frac{1}{2}\langle\overline{m_2 l_2}|\exp\left\{-\frac{1}{\sqrt{2}}A_e(\boldsymbol{\rho}_1)A_h(\boldsymbol{\rho}_2)\right\}U_{eh} \\ &\times \exp\left\{-\frac{1}{\sqrt{2}}A_h^\dagger(\boldsymbol{\rho}_2)A_e^\dagger(\boldsymbol{\rho}_1)\right\}|\overline{l_1 m_1}\rangle. \end{aligned} \tag{18}$$

Expanding the exponents and exploiting the fact that $U_{eh}(\boldsymbol{\rho}_2 - \mathbf{r})$ does not depend on $\boldsymbol{\rho}_1$, we obtain a series

$$\overline{U}^{0m_20l_2}_{0m_10l_1} = \frac{1}{2}\sum_{p=0}^{\infty}\left(\frac{1}{2}\right)^p U^{0m_2pl_2}_{0m_1pl_1}. \qquad (19)$$

Note that Eq. (19) includes contributions of the *infinitely many* LLs. For the Coulomb interactions, the matrix elements can be calculated analytically; in zero LL, we obtain

$$\begin{gathered}\left\langle \overline{m_2 l_2}\right|\bar{S}H_{\mathrm{eh}}\bar{S}^{\dagger}\left|\overline{l_1 m_1}\right\rangle \\ = \delta_{l_1-m_1,\, l_2-m_2} 2\sqrt{2}\,\overline{U}_{\min(m_1,m_2),\,\min(l_1,l_2)}(|m_1-m_2|),\end{gathered} \qquad (20)$$

$$\begin{gathered}\overline{U}_{mn}(s) = -\frac{E_0}{[m!(m+s)!n!(n+s)!]^{1/2}2^{m+n+s}3^{s+1/2}} \\ \times\sum_{k=0}^{m}\sum_{l=0}^{n} C_m^k C_n^l\left(\frac{2}{3}\right)^{k+l}[2(k+l+s)-1]!![2(m-k)-1]!! \\ \times\sum_{p=0}^{n-l} C_k^p C_{n-l}^p(-1)^p p![2(n-l-p)-1]!!.\end{gathered} \qquad (21)$$

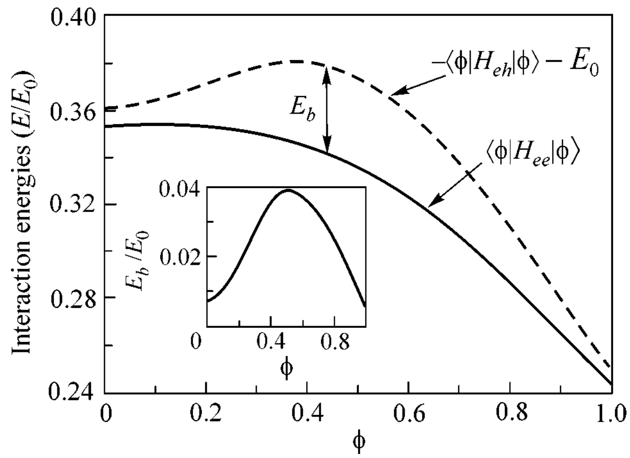
The developed formalism can be used for performing a rapidly convergent [5] expansion of the interacting e–h states in the basis (15), which preserves all symmetries of the problem. Here we demonstrate a possibility of using the squeezed states for constructing *trial* wave functions. We consider the triplet charged 2D magnetoexciton in zero LL, X^-_{t00}, with $M_z = -1$, which is the only bound state [8, 9] in zero LL in the strictly 2D system in the high-field limit. The *simplest possible* wave function in zero LL compatible with *all* symmetries of the problem is

$$\begin{gathered}\langle \mathbf{r}\mathbf{R}\mathbf{r}_h|B_e^{\dagger}(\mathbf{r})|\tilde{0}\rangle = \frac{1}{\sqrt{2}(2\pi l_B^2)^{3/2}}\left(\frac{z^*}{\sqrt{2}l_B}\right) \\ \times\exp\left(-\frac{\mathbf{r}^2+\mathbf{R}^2+\mathbf{r}_h^2-\sqrt{2}Z^*z_h}{4l_B^2}\right).\end{gathered} \qquad (22)$$

This form allows analytic calculations and, as a squeezed state (see above), already ensures the X^-_{t00} binding. Indeed, the total Coulomb interaction energy is given by

$$\frac{1}{\sqrt{2}}V_{01}^{01} + 2\sqrt{2}\,\overline{U}_{10}(0) = \left(\frac{\sqrt{2}}{4}-\frac{5\sqrt{6}}{9}\right)E_0 \simeq -1.007E_0.$$

The corresponding binding energy (counted from the ground-state energy of the neutral magnetoexciton, $-E_0$) is $0.007E_0$, which is 17% of the numerically "exact" value of $0.043E_0$ [8, 5]. A similar type of squeezing can be applied to construct a trial wave function of the X^-_{t00}. The idea is to additionally squeeze the effective hole $\boldsymbol{\rho}_2$ and electron $\mathbf{r}$ coordinates. Since the wave function must be antisymmetric under the permutation of the electron coordinates, we can use the form $|\phi\rangle \sim B_e^{\dagger}(\mathbf{r})\,(S_{\phi} + S_{-\phi})\bar{S}^{\dagger}|\bar{0}\rangle$, where the second two-mode squeezing operator is given by $S_{\phi} = \exp[\phi B_e^{\dagger}(\mathbf{r})B_h^{\dagger}(\boldsymbol{\rho}_2) - \mathrm{H.c.}]$ and we have used $|\tilde{0}\rangle = \bar{S}^{\dagger}|\bar{0}\rangle$. The normalized *four*-mode squeezed wave function has the form



The expectation values of the e–e repulsion, $\langle\phi|H_{ee}|\phi\rangle$, and the e–h attraction, $\langle\phi|H_{eh}|\phi\rangle$ (with the opposite sign, counted from the neutral magnetoexciton binding energy $E_0 = \sqrt{\pi/2}\,(e^2/\epsilon l_B)$), for the trial wave function (23) of the charged triplet magnetoexciton in zero LLs, X^-_{t00}. The binding energy $E_b = -\langle\phi|H_{eh} + H_{ee}|\phi\rangle + E_0$ is shown in the inset.

$$\begin{gathered}|\phi\rangle = \frac{1+\tanh^2\phi}{\cosh^2\phi\sqrt{1+\tanh^4\phi}}B_e^{\dagger}(\mathbf{r}) \\ \times\cosh[\tanh\phi\, B_e^{\dagger}(\mathbf{r})B_h^{\dagger}(\boldsymbol{\rho}_2)]\bar{S}^{\dagger}|\bar{0}\rangle.\end{gathered} \qquad (23)$$

The calculated energy of the Coulomb e–e repulsion, $\langle\phi|H_{ee}|\phi\rangle$, monotonically decreases with increasing transformation angle ϕ, whereas the energy of the e–h attraction, $-\langle\phi|H_{ee}|\phi\rangle$, has a maximum (see figure). The binding of the X^-_{t00} results from a rather delicate balance between the two terms, and for the state (23) the maximum achieved binding energy is $E_b \simeq 0.038E_0$ (see inset in figure), which is 91% of the "exact" value [5, 8]; note that the inaccuracy is 0.3% of the e–h interaction energy. A similar type of squeezed trial wave functions may be useful in other solid state and atomic physics problems dealing with correlated e–h states in strong magnetic fields.

In conclusion, we have developed for charged e–h systems in magnetic fields an operator approach that allows one to partially separate the CM motion. This results in the appearance of new effective particles, electrons and holes in a magnetic field, with modified interparticle interactions. A relation of the considered basis states to the two-mode squeezed oscillator states has been established.

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