Intraexcitonic transitions in two-dimensional systems in a high magnetic field

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The internal transitions of two-dimensional (2D) excitons in a high magnetic field B exhibit features due to the coupling of the internal and center-of-mass motions. A study is made of these features, and it is shown that for magnetoexcitons with a center-of-mass momentum $\mathbf{K} \neq 0$ the energies of the strong transitions decrease with increasing $\mathbf{K}$, and the absorption spectra show weakly resolved transitions, whose total intensity depends strongly on the exciton statistics (distribution function). © *1997 American Institute of Physics.* [S0021-3640(97)00421-0]

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1. Intraband transitions of quasi–two-dimensional excitons in quantum wells (QWs) and superlattices in a magnetic field have attracted a great deal of interest in recent years (see Refs. 1–3 and the literature cited therein). Progress in this field requires a sensitive method of investigation — optically detected cyclotron resonance. Intraband IR magnetospectroscopy could be effective for studying the kinetics of interlevel excitonic transitions, for investigating collective effects in a system of excitons with finite density, and for resolving the fine structure of the ground and excited states of quasi-2D excitons, for example, in coupled double QWs.[4]

In the case of intraband IR spectroscopy, all populated excitonic states give a response, including states with finite center-of-mass momentum $\mathbf{K}$. This is in contrast to interband transitions for which only excitons with $\mathbf{K}=0$ are optically active. Physically, the center-of-mass and relative motions of a neutral e–h pair are coupled in a magnetic field B. The present letter examines theoretically some characteristics of excitonic IR absorption, which are associated with this circumstance, in 2D systems in a high magnetic field. Similar effects should exist in atomic physics (taking account of the change in the characteristic magnetic field and momentum scales[5]).

2. For simplicity, we shall study the purely 2D situation. Motion of a 2D neutral e–h pair in a transverse magnetic field $\mathbf{B}=(0,0,B)$ is described by the Hamiltonian

$$H=\frac{1}{2m_e}\left(-i\hbar\boldsymbol{\nabla}_e+\frac{e}{c}\mathbf{A}_e\right)^2+\frac{1}{2m_h}\left(-i\hbar\boldsymbol{\nabla}_h-\frac{e}{c}\mathbf{A}_h\right)^2-\frac{e^2}{\epsilon|\mathbf{r}_e-\mathbf{r}_h|}\equiv H_0+U_{eh}, \qquad (1)$$

where $\mathbf{r}=(x,y)$. The motion is characterized[5] by a conserved magnetic momentum of the center of mass $\hat{\mathbf{K}}=-i\hbar\boldsymbol{\nabla}_{\mathbf{R}}-(e/c)\,\mathbf{A}(\mathbf{r})$. Here $\mathbf{R}=(m_e\mathbf{r}_e+m_h\mathbf{r}_h)/M$ are the coordinates of the center of mass and $\mathbf{r}=\mathbf{r}_e-\mathbf{r}_h$ are the relative e–h coordinates, $M=m_e+m_h$, and

$\mathbf{A}=\frac{1}{2}\mathbf{B}\times\mathbf{r}$. The wave function of an exciton with momentum $\mathbf{K}$ can be represented in the form[5] $\Psi_{\mathbf{K}}=\exp(i/\hbar)[\mathbf{K}+(e/c)\,\mathbf{A}(\mathbf{r})]\cdot\mathbf{R})\Phi_{\mathbf{K}}(\mathbf{r})$. This can also be regarded as a unitary transformation of the Hamiltonian $H\rightarrow\widetilde{H}(\mathbf{K})=\hat{U}^{\dagger}H\hat{U}$, where $\hat{U}(\mathbf{K})=\exp((i/\hbar)\times[\mathbf{K}+(e/c)\,\mathbf{A}(\mathbf{r})]\cdot\mathbf{R})$. The transformed Hamiltonian has the form[5,6] $\widetilde{H}(\mathbf{K})=\widetilde{H}_0(\mathbf{K})+U_{eh}$, and

$$\widetilde{H}_0(\mathbf{K})=-\frac{\hbar^2}{2\mu}\nabla_{\mathbf{r}}^2+\frac{1}{2}\hbar(\omega_{ch}-\omega_{ce})\hat{l}_z+\frac{e^2B^2}{8\mu c^2}r^2+\frac{e}{Mc}\mathbf{B}\cdot[\mathbf{r}\times\mathbf{K}]+\frac{K^2}{2M}, \tag{2}$$

where $\mu^{-1}=m_e^{-1}+m_h^{-1}$, $\omega_{ce(h)}=eB/m_{e(h)}c$, and $\hat{l}_z=-i[\mathbf{r}\times\nabla_{\mathbf{r}}]_z$ is the projection of the angular momentum of the relative motion. Similarly to the case of electrons in a magnetic field B, the Hamiltonian (2) can be diagonalized in a representation of Bose ladder operators (see Ref. 7). For this, we first perform another unitary transformation[5] $\widetilde{H}(\mathbf{K})\rightarrow\overline{H}=\hat{W}^{\dagger}(\mathbf{K})\widetilde{H}(\mathbf{K})\hat{W}(\mathbf{K})$, where $\hat{W}(\mathbf{K})=\exp((i/2\hbar)\,\gamma\mathbf{K}\cdot\mathbf{r})$ and $\gamma=(m_h-m_e)/M$, and then a translation of the coordinates $\mathbf{r}\rightarrow\overline{\mathbf{r}}=\mathbf{r}-\mathbf{r}_0=(\overline{x},\overline{y})$ with $\mathbf{r}_0=\mathbf{e}_z\times\mathbf{K}l_B^2/\hbar$. After this we obtain the Hamiltonian $\overline{H_0}$ (obviously, $\overline{H}=\overline{H_0}+U_{eh}(\overline{\mathbf{r}})$) which in the coordinate representation assumes the form of the Hamiltonian $\widetilde{H}_0(\mathbf{K}=0)$ from Eq. (2). To diagonalize $\overline{H_0}$ we introduce the ladder operators

$$\overline{a^{\dagger}}=\frac{1}{\sqrt{2}}\left(\frac{z}{2l_B}-2l_B\frac{\partial}{\partial z^*}\right),\qquad \overline{b^{\dagger}}=\frac{1}{\sqrt{2}}\left(\frac{z^*}{2l_B}-2l_B\frac{\partial}{\partial z}\right), \tag{3}$$

such that $[\overline{a},\overline{a^{\dagger}}]=[\overline{b},\overline{b^{\dagger}}]=1$ and $[\overline{a},\overline{b}]=[\overline{a},\overline{b^{\dagger}}]=0$; here $z=\overline{x}+i\overline{y}$ and $l_B=(\hbar c/eB)^{1/2}$. In this representation, we have $\overline{H_0}=\hbar\omega_{ce}(\overline{a^{\dagger}}\,\overline{a}+\frac{1}{2})+\hbar\omega_{ch}(\overline{b^{\dagger}}\,\overline{b}+\frac{1}{2})$, so that the orthonormalized eigenstates have the form of factorized wave functions $|nm\rangle=(\overline{a^{\dagger}})^n(\overline{b^{\dagger}})^m|00\rangle/\sqrt{n!m!}$ with eigenvalues $\hbar\omega_{ce}(n+\frac{1}{2})+\hbar\omega_{ch}(m+\frac{1}{2})$. In the coordinate representation the wave functions $\langle\mathbf{r}|nm\rangle\equiv\phi_{nm}(\mathbf{r})$ are identical to the wave functions of an electron in a field B (for example, $\langle\mathbf{r}|00\rangle=\exp(-\rho^2/4l_B^2)/(2\pi l_B^2)^{1/2}$). In the case of a magnetoexciton the operators $\overline{a^{\dagger}}$, $\overline{a}$ ($\overline{b^{\dagger}}$, $\overline{b}$) describe electronic (hole) Landau levels. Since $\hat{S}(\mathbf{K})\equiv\hat{W}(\mathbf{K})\hat{U}(\mathbf{K})=\exp((i/\hbar)\mathbf{R}_0\cdot[\mathbf{K}+(e/c)\,\mathbf{A}(\mathbf{r})])$, where $\mathbf{R}_0=\frac{1}{2}(\mathbf{r}_e+\mathbf{r}_h)$, the wave functions $|nm\mathbf{K}\rangle=\hat{S}(\mathbf{K})|nm\rangle$ describing the free motion of an e–h pair in a field B can be represented in the form

$$\Psi_{nm\mathbf{K}}(\mathbf{r}_e,\mathbf{r}_h)=\langle\mathbf{r}_e\mathbf{r}_h|nm\mathbf{K}\rangle=\exp\left(\frac{i}{\hbar}\mathbf{R}_0\cdot\left[\mathbf{K}+\frac{e}{c}\mathbf{A}(\mathbf{r})\right]\right)\phi_{nm}(\mathbf{r}-\mathbf{r}_0). \tag{4}$$

The wave functions $\Psi_{nm\mathbf{K}}(\mathbf{r}_e,\mathbf{r}_h)$ correspond in the limit of a high magnetic field (cf. Ref. 6) to 2D magnetoexcitons with the dispersion relation $E_{nm}(\mathbf{K})=\langle nm|U_{eh}(\mathbf{r}-\mathbf{r}_0)|nm\rangle$.

3. Let us examine the interaction of excitons with IR radiation. In the Faraday geometry (the radiation propagates parallel to $\mathbf{B}$) the Hamiltonian describing absorption accompanying an interaction with the ac electric field (with amplitude $\mathcal{F}_0$ and frequency ω) of circularly polarized IR radiation has the form

$$\delta\hat{V}^{\pm}=\frac{e\mathcal{F}_0}{\omega}\left(\frac{\pi_e^{\pm}}{m_e}-\frac{\pi_h^{\pm}}{m_h}\right)\exp(-i\omega t). \tag{5}$$

Here the $\pm$ signs denote left (right) circular polarization $\sigma^{\pm}$, and

$$\pi_j^{\pm}=\pi_{jx}\pm i\pi_{jy},(j=e,h), \quad \boldsymbol{\pi}_e=-i\hbar\boldsymbol{\nabla}_e+\frac{e}{c}\mathbf{A}_e, \quad \boldsymbol{\pi}_h=-i\hbar\boldsymbol{\nabla}_h-\frac{e}{c}\mathbf{A}_h.$$

It can be shown that $[\delta\hat{V}^{\pm},\hat{\mathbf{K}}]=0$, i.e., magnetic momentum is conserved in IR transitions (in the dipole approximation this also follows from the law of conservation of the total momentum). For $\mathbf{K}=0$, magnetoexcitons can be characterized by the conserved projection of the angular momentum l_z of the relative e–h motion; here $l_z=n-m$ $(\hat{l}_z=\overline{a^{\dagger}}\,\overline{a}-\overline{b^{\dagger}}\,\overline{b})$. For this reason, for excitons with $\mathbf{K}=0$ in a field B the selection rules have the standard form

$$\langle\Psi'_{\mathbf{K}=0,l'_z}|\delta\hat{V}^{\pm}|\Psi_{\mathbf{K}=0,l_z}\rangle\sim\delta_{l'_z,l_z\pm 1}. \tag{6}$$

For $\mathbf{K}\neq 0$, on account of the presence of the term $(e/Mc)\mathbf{B}\cdot[\mathbf{r}\times\mathbf{K}]$ (which corresponds to a uniform electric field in the moving coordinate system in B), the Hamiltonian (2) does not possess axial symmetry. As a result, the selection rules for IR transitions reduce to only conservation of momentum: generally speaking, $\langle\Psi'_{\mathbf{K}}|\delta\hat{V}^{\pm}|\Psi_{\mathbf{K}}\rangle\neq 0$ for all pairs of excitonic terms. The analysis simplifies in the high-field limit. The matrix elements of the operator describing the interaction with the IR radiation field between states of the 2D magnetoexcitons (4) have the form

$$\langle n'm'\mathbf{K}|\delta\hat{V}^{\pm}|nm\mathbf{K}\rangle=\langle n'm'|\hat{S}(\mathbf{K})^{\dagger}\delta\hat{V}^{\pm}\hat{S}(\mathbf{K})|nm\rangle. \tag{7}$$

The relation

$$\hat{S}(\mathbf{K})^{\dagger}\delta\hat{V}^{+}\hat{S}(\mathbf{K})=\frac{i\sqrt{2}e\hbar\mathcal{F}_0}{\omega l_B}\left(\frac{\overline{a^{\dagger}}}{m_e}-\frac{\overline{b}}{m_h}\right)e^{-i\omega t} \tag{8}$$

shows that the matrix element (7) does not depend on the momentum $\mathbf{K}$, and in this limit transitions are possible only with a change in the Landau level numbers $\Delta n(\Delta m)=1$ for $\sigma^{\pm}$ polarization. The mixing of the Landau levels is taken into account below.

4. Let us consider IR transitions between excitons with $\mathbf{K}=0$. In high magnetic fields, the $1s$ excitonic states are formed mainly by the state $|00\mathbf{K}=0\rangle$, which corresponds to the zeroth e and h Landau levels. On account of the e–h Coulomb interaction, there is also a weak $\sim l_B/a_{Be(h)}\ll 1$ $[a_{Be(h)}=\epsilon\hbar^2/m_{e(h)}e^2]$ admixing of higher Landau levels $|nn\mathbf{K}=0\rangle$. In a similar manner, the $2p^{+}(2p^{-})$ excitonic states are formed in the ground state $|10\mathbf{K}=0\rangle$ $(|01\mathbf{K}=0\rangle)$ with a weak admixing of the states $|n+1\,n\mathbf{K}=0\rangle$ $(|nn+1\,\mathbf{K}=0\rangle)$. For this reason, the excitonic transition $1s\rightarrow 2p^{+}$ $(1s\rightarrow 2p^{-})$ in a high field B can be regarded[4] as an electron (hole) cyclotron resonance $\phi_{00}\rightarrow\phi_{10}$ $(\phi_{00}\rightarrow\phi_{01})$, which is modified by excitonic effects. In the purely 2D case and in the limit of a high magnetic field, the binding energies of $1s$ and $2p^{\pm}$ magnetoexcitons are equal to[6] $E_{00}=E_0$ and $E_{10}=E_{01}=\frac{1}{2}E_0$, respectively; here $E_0=\sqrt{\pi/2}e^2/\epsilon l_B\sim\sqrt{B}$. For this reason, the $1s\rightarrow 2p^{\pm}$ transition energies in this limit are

$$E_{1s\to 2p^+}=\hbar\omega_{ce}+\frac{1}{2}E_0, \quad E_{1s\to 2p^-}=\hbar\omega_{ch}+\frac{1}{2}E_0. \tag{9}$$

The transitions $1s\to np^{\pm}$ to higher-lying excited states are weak $\sim[l_B/a_{Be(\mathrm{h})}]^2$, and their energies

$$E_{1s\to np^{\pm}}=\hbar\omega_{ce(h)}+(n-1)[\hbar\omega_{ce}+\hbar\omega_{ch}]+\left[1-\frac{[2(n-1)]!}{2^{2(n-1)}[(n-1)!]^2}\right]E_0 \tag{10}$$

contain a contribution which is a multiple of the sum of the e and h cyclotron energies $[\hbar\omega_{ce}+\hbar\omega_{ch}]$; the last term $\approx[1-(\pi n)^{-1/2}]E_0$ for $n\gg 1$ in Eq. (10).

Evidently, the excitonic IR transitions are sensitive to the e–h Coulomb interactions. Kohn's theorem[8] is inapplicable in this situation, since the charge-to-mass ratios are different for e and h. However, as one can see from Eq. (10), the *difference*

$$E_{1s\to np^+}-E_{1s\to np^-}=\hbar\omega_{ce}-\hbar\omega_{ch} \tag{11}$$

does not depend on the e–h interactions.[4] The result (11) follows from the fact that the variables in Eq. (2) are separable in cylindrical coordinates, and it is valid not only in the limit of a high magnetic field or for a 2D system. This can likewise be attributed to the existence of an exact symmetry for excitons in a uniform field B. To show this, we introduce[9] the time-reversal operator $\hat{T}$ which operates *only* on the system under study. The field B is assumed to be an external field: The direction of **B** does not change under the operation $\hat{T}$ (the currents generating B do not change direction). In the standard manner, the coordinates do not change sign under the operation $\hat{T}$: $\hat{T}^{-1}\mathbf{r}\hat{T}=\mathbf{r}$, while the momenta and orbital angular momenta do change sign: $\hat{T}^{-1}\hat{\mathbf{p}}\hat{T}=-\hat{\mathbf{p}}$ and $\hat{T}^{-1}\hat{\mathbf{l}}\hat{T}=-\hat{\mathbf{l}}$. For the total Hamiltonian $\widetilde{H}(\mathbf{K})=\hat{U}^{\dagger}(\mathbf{K})H\hat{U}(\mathbf{K})$, corresponding to the internal motion of an e–h pair (see Eq. (2)), we have

$$[\widetilde{H}(\mathbf{K}),\hat{T}]=[\widetilde{H}_0(\mathbf{K}),\hat{T}]=(\hbar\omega_{ch}-\hbar\omega_{ce})\hat{T}\hat{l}_z. \tag{12}$$

We shall now take into account that for excitons with $\mathbf{K}=0$ the projection l_z is a good quantum number and that $\hat{T}\Psi_{\mathbf{K}=0np^+}=\Psi_{\mathbf{K}=0np^-}$. (We note that $\hat{T}^{-1}\overline{a^{\dagger}}\hat{T}=\overline{b^{\dagger}}$, so that $\hat{T}|nm\rangle=|mn\rangle$, and the last equality is obvious in the high-field limit.) Therefore relation (11) follows from the operator algebra (12). In order for the relation (12) to hold formally it is important that the operator $\hat{T}$ is antiunitary, so that $\hat{T}^{-1}\hat{U}(\mathbf{K})\hat{T}\neq\hat{U}(\mathbf{K})$ and $[\hat{U}(\mathbf{K}),\hat{T}]\neq 0$. The analysis based on an operator algebra similar to the algebra (12) could be helpful for investigating more complicated Hamiltonians in a field B (compare with the theorem for a one-component many-electron system[9]).

5. Let us establish the characteristic features due to IR absorption by (for example, thermally excited) magnetoexcitons with $\mathbf{K}\neq 0$. We assume that the magnetic field B is high enough ($l_B\ll a_{Be(h)}$) that the mixing of different Landau levels can be taken into account by perturbation theory. The results should also be applicable qualitatively for lower fields $l_B\leq a_{Be(h)}$. We shall study the magnetic quantum limit $\nu_X=2\pi l_B^2 n_X\ll 1$, when magnetoexcitons fill the zeroth Landau levels; n_X is the exciton density.

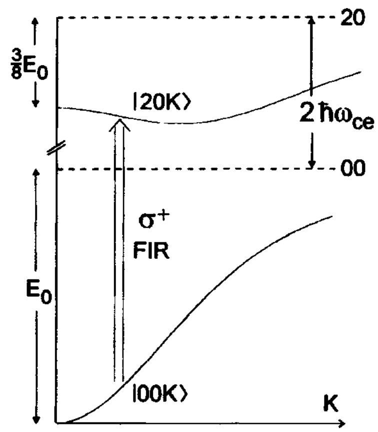

FIG. 1. Schematic illustration of the dispersion $E_{00}(K)$ and $E_{20}(K)$ of 2D magnetoexcitons $|00\mathbf{K}\rangle$ and $|20\mathbf{K}\rangle$. The vertical double arrow shows the weakly resolved σ^+ IR transition. The dashed lines mark the positions of the unoccupied Landau levels $(n_e n_h)$.

Let us consider first how the energy of a strong transition $|00\mathbf{K}\rangle \rightarrow |10\mathbf{K}\rangle$ depends on K. Assuming low temperatures $k_BT \ll E_0$, we can limit the analysis to low momenta $Kl_B/\hbar \ll 1$. The dispersion relations for magnetoexcitons in this region are quadratic:[6]

$$E_{00}(K) \simeq -E_0 + K^2/2M_{00}, \quad E_{10}(K) \simeq -\frac{1}{2}E_0 + K^2/2M_{10}, \tag{13}$$

where $M_{00} = 2\hbar^2/E_0 l_B^2$ and $M_{10} = -2M_{00}$. The magnetoexciton $|10\mathbf{K}\rangle$ is characterized by a *negative* effective mass. As a result of this, the ''kinetic'' energies of the initial and final states do not compensate each other, and the transition energy

$$E_{00\rightarrow 10} = \hbar\omega_{ce} + \frac{1}{2}E_0 - \frac{K^2}{2M_{00}}\left(1 + \frac{M_{00}}{|M_{10}|}\right) \tag{14}$$

decreases with increasing momentum K. (A similar situation for the transition $|00\mathbf{K}\rangle \rightarrow |20\mathbf{K}\rangle$ is shown in Fig. 1.) Therefore it can be expected that as the temperature increases in a high magnetic field, the line due to this transition will broaden predominantly into the region of *lower* energies. Since the dispersion of 2D magnetoexcitons is due to only *e–h* interactions,[6] this effect is simply due to the influence of interparticle interactions on intraband excitonic IR transitions.

Let us now estimate the characteristic size of the third term in Eq. (14). Since 2D magnetoexcitons form an almost ideal gas,[10] we propose for them a Bose distribution function $f_X = (\exp[(\epsilon_K - \mu)/k_BT] - 1)^{-1}$, where $\epsilon_K = K^2/2M_{00} = E_0 K^2 l_B^2/4\hbar^2$ from Eq. (13), and the chemical potential of a 2D ideal Bose gas is given by the expression $\mu = k_BT \ln[1 - \exp(-E_0\nu_X/2k_BT)]$. Therefore, for 2D magnetoexcitons, the particular regime which is realized is determined by the parameter $\zeta \equiv E_0\nu_X/k_BT$. In the classical limit, $\zeta \ll 1$, we have Maxwell–Boltzmann statistics, and $\langle K^2\rangle/2M_{00} = k_BT$. In the degenerate quantum limit, $\zeta \gg 1$ (when the chemical potential $\mu = -k_B T e^{-\zeta/2}$ is exponentially small), we obtain $\langle K^2 \rangle / 2M_{00} = \pi^2 k_B T / 3\zeta \ll k_B T$, i.e., narrowing of the absorption line occurs.

Another feature associated with IR absorption by magnetoexcitons with $\mathbf{K} \neq 0$ is due to mixing of different Landau levels. In the high magnetic field limit, when mixing is neglected, the magnetoexciton wave functions $|nm\mathbf{K}\rangle$ are given by expression (4). In the next order in the parameter $l_B / a_{Be(h)} \ll 1$, the wave functions assume the form $|\widetilde{nm}\mathbf{K}\rangle = \Sigma_{n'm'} A^{(nm)}_{n'm'} |n'm'\mathbf{K}\rangle$, where the coefficients $A^{(nm)}_{nm} = \mathcal{O}(1)$ and

$$A^{(nm)}_{n'm'} = \frac{U^{n'm'}_{nm}(\mathbf{K})}{\hbar\omega_{ce}(n-n') + \hbar\omega_{ch}(m-m')} \sim \frac{l_B}{a_{Be(h)}} \ll 1. \tag{15}$$

Here $U^{n'm'}_{nm}(\mathbf{K}) = \langle n'm'\mathbf{K}|U_{eh}|nm\mathbf{K}\rangle$ is the Coulomb matrix element between two magnetoexcitonic states. An analytical expression for $U^{n'm'}_{nm}(\mathbf{K})$ with arbitrary indices is obtained in Ref. 11 (see also Ref. 6). For magnetoexcitons with $\mathbf{K} \neq 0$ the *e–h* interaction mixes *all* states on different Landau levels $|nm\mathbf{K}\rangle$. This gives rise to a number of new lines in the spectra [cf. Eq. (6)]: $\langle \widetilde{nm}\mathbf{K}|\delta\hat{V}^{\pm}|\widetilde{00}\mathbf{K}\rangle \neq 0$. However, all transitions with $|n-m| \neq 1$ are found to be weak, of order $\sim (l_B / a_{Be(h)})^2$. Furthermore, for $k_B T \ll E_0$, the larger the difference $|n-m|$, the weaker the transition is. Let us consider as an example the transition $|\widetilde{00}\mathbf{K}\rangle \rightarrow |\widetilde{20}\mathbf{K}\rangle$ (see Fig. 1). We underscore that for $\mathbf{K}=0$ this is a strictly forbidden transition $1s \rightarrow 3d^+$. The total intensity of the transition $|\widetilde{00}\mathbf{K}\rangle \rightarrow |\widetilde{20}\mathbf{K}\rangle$ (the total absorbed power is $\simeq 2\hbar\omega_{ce} R_{20}$)

$$R_{20} = \frac{2\pi}{\hbar} \sum_{\mathbf{K}} |\langle \widetilde{20}\mathbf{K}|\delta\hat{V}^{+}|\widetilde{00}\mathbf{K}\rangle|^2 f_X(K,T) \tag{16}$$

depends on the population of different $\mathbf{K}$ states; $f_X(K,T)$ is the Bose distribution function of 2D magnetoexcitons in the zeroth Landau level. In the classical ($\zeta \ll 1$) and quantum ($\zeta \gg 1$) limits we obtain

$$R_{20} = \frac{25}{32} \frac{e^2 \mathcal{F}_0^2}{\hbar} \zeta \left(\frac{k_B T}{\hbar\omega_{ce}} \right)^2 \sim T B^{-5/2}, \quad \zeta \equiv E_0 \nu_X / k_B T \ll 1 , \tag{17}$$

$$R_{20} = \frac{25}{32} \frac{e^2 \mathcal{F}_0^2}{\hbar} \left[\frac{\pi^2}{3} - \zeta e^{-\zeta/2} \right] \left(\frac{k_B T}{\hbar\omega_{ce}} \right)^2 \sim T^2 B^{-2}, \quad \zeta \gg 1 . \tag{18}$$

It is interesting to note that in the classical limit (17) the total intensity $R_{20} \sim n_X$, whereas in the quantum limit (18) R_{20} saturates and (to within exponential corrections) is independent of the exciton density n_X. Transitions to higher Landau levels $|\widetilde{00}\mathbf{K}\rangle \rightarrow |\widetilde{nm}\mathbf{K}\rangle$ (i.e., transitions in the σ^+ polarization with $N \equiv n-m > 1$) are suppressed even more strongly at low temperatures: For example, for $\zeta \ll 1$ their total intensity is

$$R_{nm} \sim \frac{\nu_X (k_B T)^{N-1}}{[(n+m-1)\hbar\omega_c]^2 E_0^{N-3}} \sim T^{N-1} B^{-(N+3)/2}, \tag{19}$$

where we have set as a simplification $\omega_{ce} \simeq \omega_{ch} = \omega_c$.

6. In summary, we have studied the internal magnetooptic transitions of 2D excitons. It was established that for excitons with center-of-mass momentum $\mathbf{K}=0$ the spectra contain pairs of transitions differing in energy by the difference of the cyclotron energies of an electron and hole $\hbar(\omega_{ch}-\omega_{ce})$. This result was obtained for the case of simple bands with quadratic dispersion relations. A recent experiment[12] showed that this property also holds approximately for quasi-2D excitons in a GaAs/GaAlAs quantum well with a complicated valence band. This situation will be studied theoretically in a separate publication. It was predicted that for magnetoexcitons with $\mathbf{K}\neq 0$ the spectra of strong transitions will broaden into the region of low energies with increasing temperature. It was also shown that transitions for which $\mathbf{K}\neq 0$ and which are weakly resolved are sensitive to the magnetoexciton statistics. Thus a study of transitions of this kind could be helpful in the investigation of the condensation of 2D magnetoexcitons.

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[1] M. Salib, H. A. Nickel, G. H. Herold *et al.*, Phys. Rev. Lett. **77**, 1135 (1996).
[2] J. Cerne, J. Kono, M. S. Sherwin *et al.*, Phys. Rev. Lett. **77**, 1131 (1996).
[3] A. A. Dremin, V. B. Timofeev, D. Birkedal, and J. Hvam, *Proceedings OECS-5*, Göttingen, 1997, to be published in Phys. Status Solidi A.
[4] A. B. Dzyubenko and A. L. Yablonskii, JETP Lett. **64**, 213 (1996).
[5] L. P. Gor'kov and I. E. Dzyaloshinskiĭ, Zh. Éksp. Teor. Fiz. **53**, 717 (1967) [Sov. Phys. JETP **26**, 449 (1967)].
[6] I. V. Lerner and Yu. E. Lozovik, Zh. Éksp. Teor. Fiz. **78**, 1167 (1980) [Sov. Phys. JETP **51**, 588 (1980)].
[7] J. E. Avron, I. W. Herbst, and B. Simon, Ann. Phys. (N.Y.) **114**, 431 (1978).
[8] W. Kohn, Phys. Rev. **123**, 1242 (1961).
[9] A. B. Dzyubenko and A. Yu. Sivachenko Phys. Rev. B **48**, 14690 (1993).
[10] I. V. Lerner and Yu. E. Lozovik, Zh. Éksp. Teor. Fiz. **80**, 1488 (1981) [Sov. Phys. JETP **53**, 763 (1981)]; A. B. Dzyubenko and Yu. E. Lozovik, Fiz. Tverd. Tela (Leningrad) **25**, 1519 (1983) [Sov. Phys. Solid State **25**, 874 (1983)]; Fiz. Tverd. Tela (Leningrad) **26**, 1540 (1984) [Sov. Phys. Solid State **26**, 938 (1984)].
[11] M. Graf, P. Vogl, and A. B. Dzyubenko, Phys. Rev. B **54**, 17003 (1996).
[12] H. A. Nickel, G. H. Herold, M. S. Salib *et al.*, *Proceedings EP2DS-12*, Tokyo, 1997, to be published in Physica B.

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