Thermal conductivity of a metal

Wiedemann-Franz law, empirical (1853)

$$\frac{\text{thermal conductivity}}{\text{electrical conductivity}} = \frac{\kappa}{\sigma} \sim T$$

$\frac{\kappa}{\sigma T}$, Lorenz number is the same (to good accuracy) for several metals

at $T = 373\,K$:	Cu	Ag	Au	Li	$\frac{\text{Watt-ohm}}{K^2}$
	2.29	2.38	2.36	2.43	$\times 10^{-8}$

Drude theory: internal energy flow is mainly due to electrons

$\kappa = \kappa_e \gg \kappa_{ions}$ (phonon mechanism) is less efficient

$$\vec{j}^{\,q} = -\kappa \vec{\nabla} T$$

"thermal current" density = internal energy density flow

$$\ell = v\tau$$

$$\boxed{\kappa = \frac{1}{3} v^2 \tau c_v} = \frac{1}{3} v \ell c_v$$

<u>electron</u> values
not lattice

$$\frac{\kappa}{\sigma} = \frac{\frac{1}{3} v^2 \tau c_V}{\frac{ne^2\tau}{m}} = \frac{1}{3} \cdot \frac{mv^2}{ne^2} \cdot c_V$$

τ is gone...

Drude: electrons as classical gas in thermal equilibrium at temperature T

$$\frac{mv^2}{2} = \frac{3}{2} k_B T$$

: classical equipartion

$$c_V = \frac{3}{2} k_B \longrightarrow \text{per one electron}$$

$$\frac{\kappa}{\sigma} = \frac{1}{3} \cdot \frac{3k_B T}{ne^2} \cdot \frac{3}{2} k_B \cdot n = \underbrace{\frac{3}{2}\left(\frac{k_B}{e}\right)^2}_{\text{Lorenz number}} \cdot T$$

very reasonable! agreement with experiment

Lorenz number $1.11 \times 10^{-8} \frac{\text{Watt-ohm}}{K^2}$

- No electronic contribution $c_V = \frac{3}{2} k_B n$ observed ?!

Δ $c_V \sim 10^{-2} c_V$ classical

∇ $v \sim 10^2 v$ classical

two errors compensate

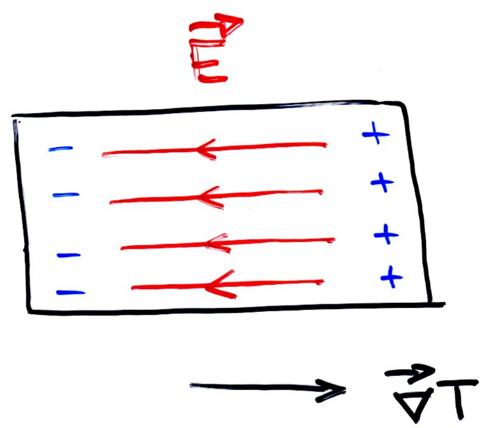
22.14

Thermoelectric effects

higher T $\longrightarrow$ higher velocities v

$\Rightarrow$ electrons tend to move out of the region with high T

Steady-state

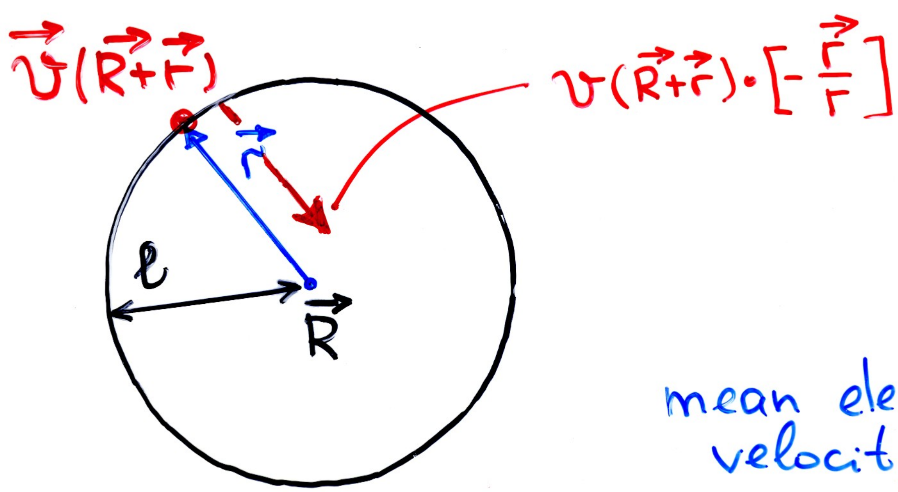


Seebeck effect

$$\vec{E} = Q \, \vec{\nabla} T$$

thermopower

Simple estimate of Q



mean electronic velocity

$$\vec{V}_Q(\vec{R}) = \int_{|\vec{r}|=\ell} \frac{d\Omega}{4\pi} \, v(\vec{R}+\vec{r})\cdot\left[-\frac{\vec{r}}{r}\right]$$

$$v(\vec{R}+\vec{r}) = v(\vec{R}) + (\vec{r}\cdot\vec{\nabla}_R)\, v(\vec{R}) + \dots$$

$$\vec{V}_Q = -\underbrace{\int_{|\vec{r}|=\ell} \frac{d\Omega}{4\pi} \left(\frac{r_i r_j}{r^2}\right)}_{\frac{1}{3}\delta_{ij}} \cdot \ell\, \hat{e}_i \frac{\partial v(\vec{R})}{\partial R_j}$$

$\ell = v(R)\tau$ + corrections $|\vec{\nabla} v| \cdot \ell$

$$\vec{V}_Q = -\frac{1}{3}\tau v(\vec{R})\, \vec{\nabla}_R v(\vec{R}) \rightarrow -\frac{1}{6}\tau \vec{\nabla}_{\vec{R}} v^2(\vec{R})$$

$$\boxed{\vec{V}_Q = -\frac{\tau}{6}\frac{d v^2(\vec{R})}{dT} \cdot \vec{\nabla}_R T}$$

Drift in $\vec{E}$: $\vec{V}_E = -\dfrac{e\vec{E}\tau}{m}$

Closed

~~Open~~-circuit: net current $= 0$

$$\vec{V}_Q + \vec{V}_E = 0 \qquad \vec{E} = Q\vec{\nabla} T$$

specific heat

$$Q = -\frac{1}{3e}\frac{d}{dT}\left(\frac{mv^2}{2}\right) = -\frac{1}{3e}\cdot\frac{c_V}{n}$$

thermopower

electron density

22.16

Drude: classical equipartition $C_V = \frac{3}{2} n k_B$

thermo-power $$\boxed{Q = -\frac{k_B}{2e} = -0.43\times10^{-4}\ \frac{\text{Volt}}{K}}$$

- does not depend on τ
- ~100 overestimation !??
- in some metals the sign of Q is positive (resembles R_H)

$\Longrightarrow$ inadequacy of classical consideration $\Longrightarrow$

Proper use of quantum statistics...

The Zommerfeld theory of metals

$\boxed{\text{Pauli exclusion principle}}$

$\boxed{\text{Fermi-Dirac, not Boltzmann distribution function for } e}$

Maxwell-Boltzmann distribution

$$f_B(\vec{v}) = A\exp\left(-\frac{m\vec{v}^2}{2k_BT}\right)$$

Number of electrons per unit volume with velocities $\vec{v} \div \vec{v}+d\vec{v}$: $f_B(\vec{v})\,d\vec{v}$

$$n = \int d\vec{v}\; A\exp\left(-\frac{m\vec{v}^2}{2k_BT}\right)$$

electron density

$$\boxed{A = n\left(\frac{m}{2\pi k_BT}\right)^{3/2}}$$

Pauli exclusion principle $\longrightarrow$ Fermi-Dirac distribution function

$$f(\vec{v}) = B\cdot\frac{1}{\exp\left[\left(\frac{mv^2}{2} - E_F\right)/k_BT\right]+1}$$

μ

"Fermi energy"

determined by normalization

$$\int d\vec{v}\, f(\vec{v}) = n$$

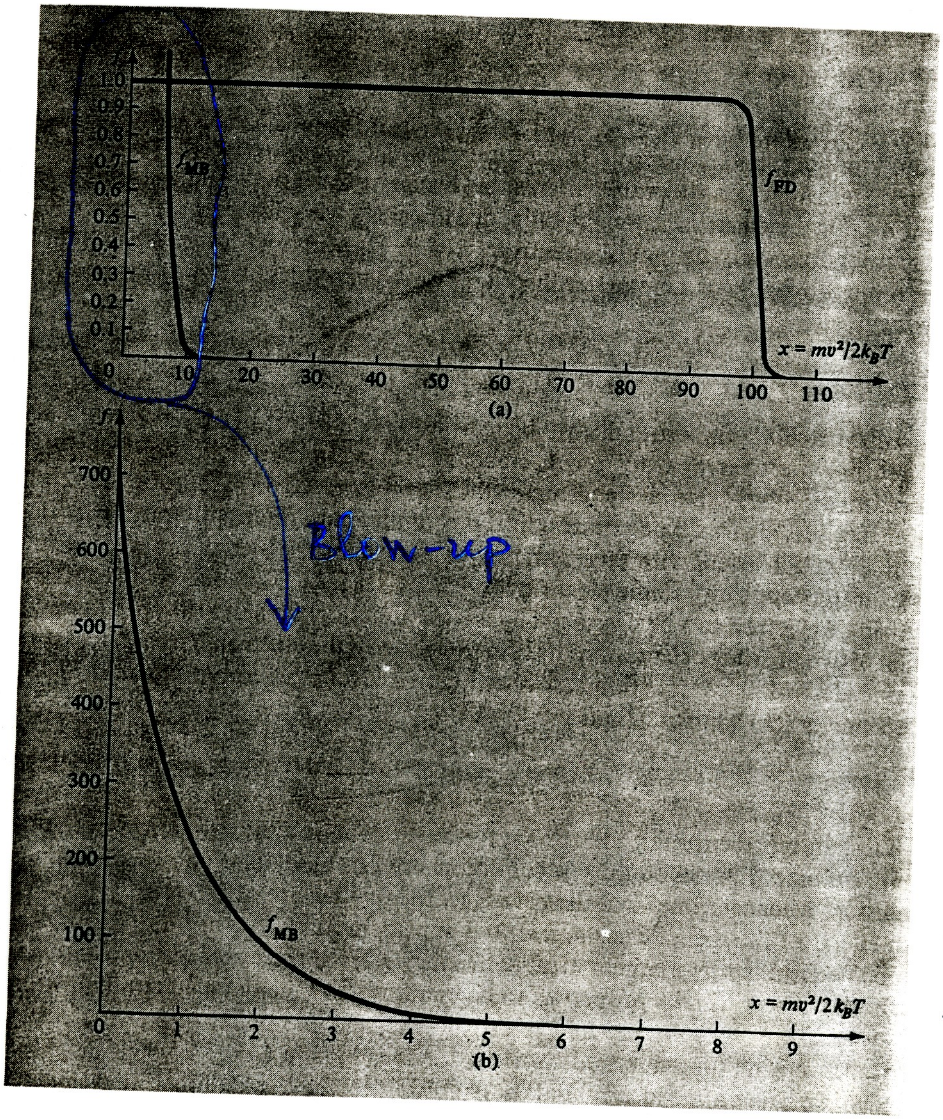
$$dN = 2\cdot\frac{d\vec{p}\,d\vec{r}}{(2\pi\hbar)^3}$$

spins up↑ down↓

$$dn = \frac{2m^3\,d\vec{v}}{(2\pi\hbar)^3}$$

$$\boxed{B = \frac{(m/\hbar)^3}{4\pi^3}}$$

22.18



$$\frac{mv^2}{2k_BT}$$

$k_BT_0 = E_F$ shown for $T = 10^{-2}T_0$

$E_F \sim$ few eV $\Rightarrow T_0 \sim 10^4 - 10^5$ K

22.19