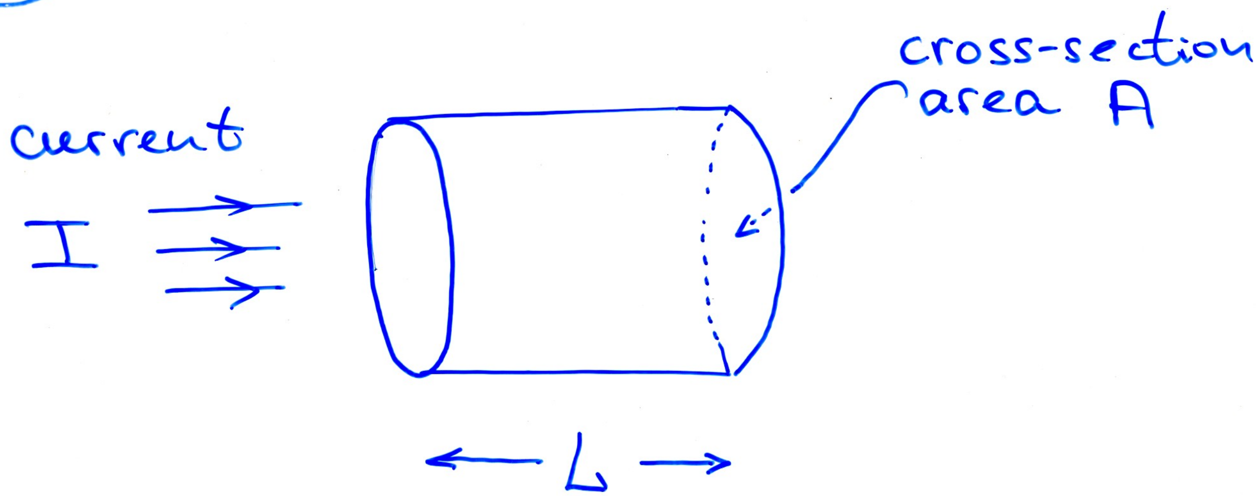
Resistance in different D

3D



voltage drop $V = E \cdot L$

Ohm's law $V = I \cdot R$

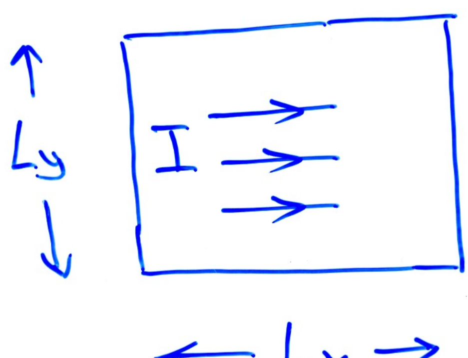
$I = j \cdot A$ current density

$j = \sigma E = \frac{1}{\rho} E$

conductivity, resistivity: characteristics of a material

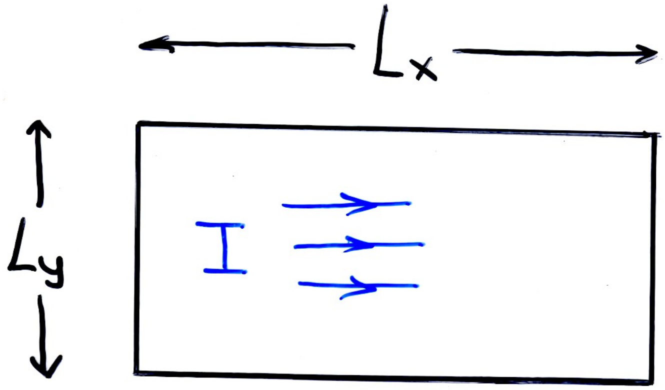
Resistance $R = \rho \boxed{\frac{L}{A}}$: sample geometry is involved

2D



how is geometry involved?

$$\boxed{R = \rho L^{2-D}, \text{ a hypercube}}$$



$V = E \cdot L_x$ voltage drop

current density in 2D $[j] = A/cm$

$$j = \frac{I}{L_y} = \sigma E \Rightarrow I = \sigma E \cdot L_y$$

$$V = E \cdot L_x = I \cdot R = \sigma E \cdot L_y \cdot R$$

$$R = \underbrace{\frac{1}{\sigma}}_{\text{inverse conductivity}} \cdot \frac{L_x}{L_y} = \rho \cdot \frac{L_x}{L_y}$$

$$\boxed{\text{in 2D: resistivity } \rho = R_\square}$$

square ($L_x = L_y$)

$R_\square$: resistance per square

2D: current density

$$\vec{j} = ne\vec{v}$$

n: areal $[cm^{-2}]$ density of particles

Basics of Hall Effect

revisited

$$\begin{cases} \sigma_0 E_x = \omega_c \tau \cdot j_y + j_x \\ \sigma_0 E_y = -\omega_c \tau \cdot j_x + j_y \end{cases}$$

$\sigma_0 = \dfrac{ne^2\tau}{m}$ Drude conductivity

$\omega_c = \dfrac{eB}{mc}$ cyclotron frequency or Larmor

$\vec{E} = \hat{\rho}\,\vec{j}$ resistivity tensor in $\vec{B}$

$$\hat{\rho} = \begin{pmatrix} \frac{1}{\sigma_0} & \frac{\omega_c\tau}{\sigma_0} \\ -\frac{\omega_c\tau}{\sigma_0} & \frac{1}{\sigma_0} \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}$$

$\rho_{xx}(\vec{B}) = \rho_{xx}(-\vec{B})$

$\rho_{xy}(\vec{B}) = \rho_{yx}(-\vec{B})$

symmetry relations of kinetic coefficients

$\rho_{xy}(\vec{B}) = -\rho_{yx}(\vec{B})$ odd function of $\vec{B}$

$\rho_{xx}(\vec{B}) = \rho_{xx}(-\vec{B})$ even function of $\vec{B}$

$$\vec{j} = \hat{\sigma}\vec{E}$$ conductivity tensor,

$$\hat{\sigma} = \hat{\rho}^{-1}$$

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

$$\hat{\sigma} = \begin{pmatrix} \dfrac{\sigma_0}{1+(\omega_c\tau)^2} & \dfrac{\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2} \\ -\dfrac{\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2} & \dfrac{\sigma_0}{1+(\omega_c\tau)^2} \end{pmatrix}$$

① calculate

② express $\sigma_{\alpha\beta}$ in terms of $\rho_{\gamma\delta}$ explicitly $\sigma_{xx} = \dots$

③ show that if $\rho_{xx} \to 0$ resistivity vanishes (2D), then $\sigma_{xx} \to 0$ conductivity vanishes too.

22.3 b

Quantum Hall Effect

integer:
(K. von Klitzing et al. 1980)

2D systems

Strong fields B:

$$\hbar\omega_c =: \frac{\hbar^2}{m \ell_B^2} \Rightarrow \ell_B = \left(\frac{\hbar c}{eB}\right)^{1/2}$$

magnetic length

$\ell_B \simeq 81\,\mathring{A}$

$B = 10T = 10^5 G$

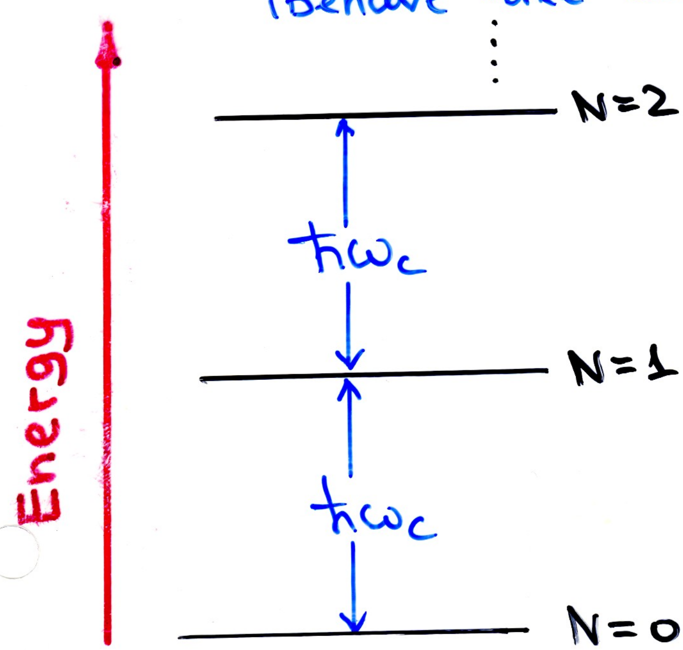
$$\hbar\omega_c \gtrsim \frac{e^2}{a_0} \Longleftrightarrow a_0 = \frac{\epsilon \hbar^2}{m e^2} \gtrsim \ell_B$$

Bohr radius (effective)

$N = 0, 1, 2, \ldots$

Free electrons in B: Landau levels
(Behave "like" oscillators)

each is **Macroscopically** degenerate



$$N_0 = \frac{S}{2\pi \ell_B^2} \sim B_\perp$$

S: area of the sample $\perp \vec{B}$

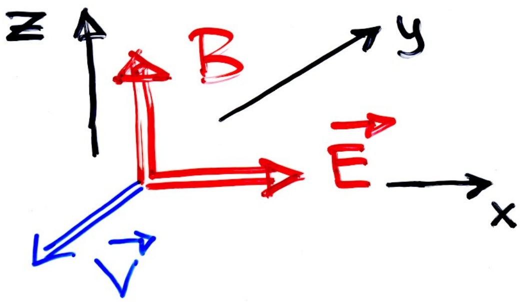
22.3c

Motion of a free! charge in $\vec{E} \perp \vec{B}$

crossed fields

Drift with velocity $\vec{V} = c \dfrac{\vec{E} \times \vec{B}}{B^2}$

direction independent of the charge sign



Go to the moving frame!

2D current density

$$j_y = +ne V_y = +ne \cdot c \frac{E_x}{B} = \sigma_{yx} E_x$$

areal electron density

conductivity tensor

Suppose we have $m = 1, 2, 3, ..$ filled Landau levels

$$n = m \cdot \frac{1}{2\pi l_B^2} \quad , \quad m \text{ integer}$$

$$E_x = \rho_{xy} j_y \leftrightarrow R_{xy}^{\square} = \frac{B}{m \cdot \frac{1}{2\pi l_B^2} \cdot ec}$$

resistivity tensor

$m = 1, 2, \ldots$

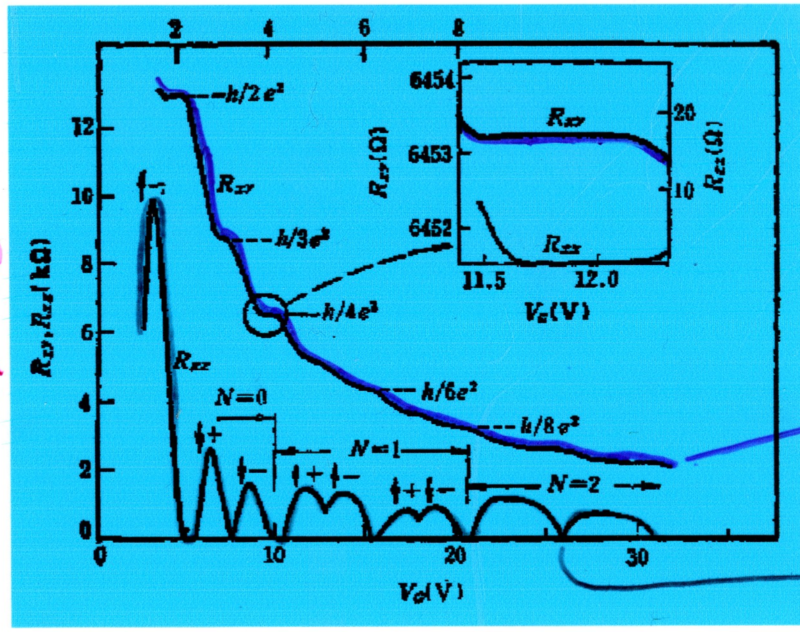
quantized value

$$\boxed{R_{xy}^{\square} = \frac{1}{m} \cdot \frac{h}{e^2}}$$

$h = 2\pi\hbar$

longitudinal resistance

only fundamental constants involved!



Rxx, Rxy

at low T

$\frac{h}{e^2} \approx 25\,\mathrm{k}\Omega$

Rxy quantized

Rxx vanishes

# of filled LLs increases

Fig.5 IQHE observed in Si-MOSFET

Integer QHE
$R_{xy} = \frac{1}{m} \cdot \frac{h}{e^2}$
$m = 1, 2, 3, \ldots$

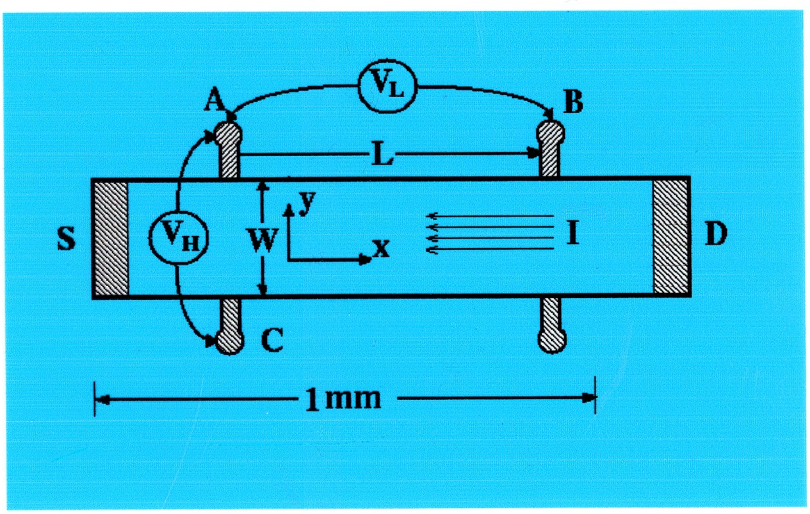


Fig.6 Experiment setup

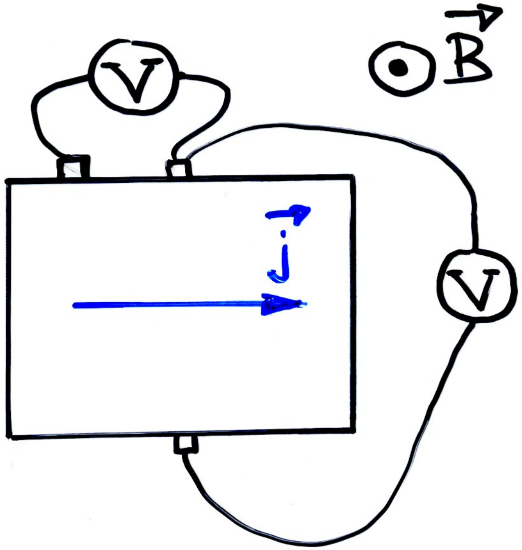
$R_{xy} = \frac{1}{m} \cdot \frac{h}{e^2}$

The experiments show that between two adjacent Landau levels, the Hall resistance has fixed values and the longitudinal resistance R_{xx} vanishes, which means that the electrons are localized in this region. Localization is a key point to interpret IQHE.

Due to impurity, the density of states will evolve from sharp Landau levels to a broader spectrum of levels(Figure 7). There are two kinds of levels , localized and extended , in the new spectrum , and it is expected that the extended states occupy a core near the orighinal Landau level energy while the localized states are more spread out in energy. Only the extended states can carry current at zero temperautre. Therefore, if the occupation of the extended states does not change, neither will the current change. An argument due to Laughlin(1981) and Halperin(1982) shows that extended states indeed exist at the cores of the Landau levels and if these states are full, (i.e., the Fermi level is not in the core of extended states) then they carry exactly the right current to give Eq. (22).

The Fracti[o]nal Hall Effect (FQH[E])

2D electron gas



$\vec{j} = -n e \vec{v}$

areal (cm^{-2}) density

$E^{\mu} = \rho_{\mu\nu} j^{\nu}$

$R_H \leftrightarrow \rho_{xy} = -\frac{1}{\nu}\frac{h}{e^2}$

$R \leftrightarrow \rho_{xx} \to 0$

$\nu = \frac{p}{q} \leftarrow$ odd

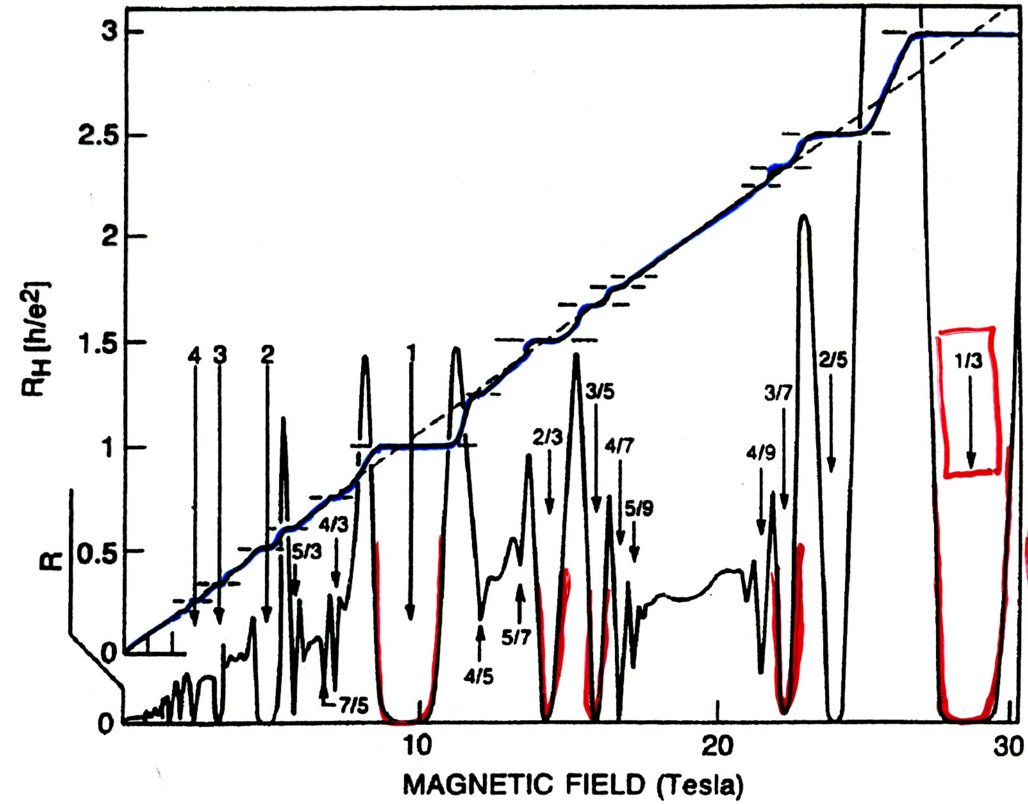


Figure 1.2: Integer and fractional quantum Hall transport data showing the plateau regions in the Hall resistance R_H and associated dips in the dissipative resistance R. The numbers indicate the Landau level filling factors at which various features occur. After ref. [15].

ρ_{xx}

Experiment: H. Störmer and D. Tsui, 1982

Microscopic Theory: R. Laughlin, 1983

Nobel Prize in Physics 1998

AC electrical conductivity $\sigma(\omega)$

$$\vec{E} = \vec{E}(t) = \mathrm{Re}\left[\vec{E}_\omega \cdot e^{-i\omega t} \right]$$

when equations are linear in $\vec{E}$ $\Rightarrow$ we can forget about taking Re ... until the very end

Equation of motion $\quad \dfrac{d\vec{p}}{dt} = -e\vec{E} - \dfrac{\vec{p}}{\tau}$

Steady-state solutions

$$\vec{p} = \vec{p}(t) = \vec{p}_\omega \cdot e^{-i\omega t}$$

complex amplitude, as $\vec{E}_\omega$ above

$$-i\omega \vec{p}_\omega = -e\vec{E}_\omega - \frac{\vec{p}_\omega}{\tau} \qquad \vec{p}_\omega = \frac{-e\vec{E}_\omega}{\frac{1}{\tau} - i\omega}$$

$$\vec{j} = \vec{j}(t) = -\frac{ne\vec{p}(t)}{m}$$

$$\vec{j} = \vec{j}_\omega e^{-i\omega t} \qquad \boxed{\vec{j}_\omega = \sigma(\omega)\vec{E}_\omega}$$

$$\boxed{\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}} \qquad \sigma_0 = \frac{ne^2\tau}{m}$$

AC $\sigma(\omega) \longrightarrow$ apply to propagation of EM waves ?

1. $\vec{E}(\vec{r},\omega)$ $\underline{\vec{B}(\vec{r},\omega)}$?

2. $\vec{r}$- dependence ?

action of B $\sim \frac{v}{c} \ll 1$

$$\vec{j}(\vec{r},\omega) = \sigma(\cancel{\vec{r}},\omega)\, E(\vec{r},\omega)$$

local dependence

good approximation when $\lambda = \frac{2\pi}{\omega} c \gg \ell$

electron mean free path

not too high frequencies ω

[$\lambda \sim 10^3 - 10^4$ Å visible light]

Maxwell's equations

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho = 0$$

induced charge density, $\rho=0$ considered.

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

current density

$\mu = 1$

$\vec{B} = \mu\vec{H} = \vec{H}$

non-magnetic media considered

22.7

... To get the wave equation

1). t-dependence $e^{-i\omega t} \cdot A_\omega(\vec{r})$

2). $\vec{j}_\omega = \sigma(\omega)\vec{E}_\omega$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\times\vec{B} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}\right)$$

$$\text{rhs} = \frac{i\omega}{c}\cdot\frac{4\pi}{c}\sigma(\omega)\vec{E} - \left(\frac{i\omega}{c}\right)^2\vec{E}$$

$$\text{lhs} = \left(\vec{\nabla}\times(\vec{\nabla}\times\vec{E})\right) = \vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{0}) - \nabla^2\vec{E}$$

$$\boxed{-\nabla^2\vec{E} = \left(\frac{\omega}{c}\right)^2 \underbrace{\left[1 + \frac{4\pi i\sigma(\omega)}{\omega}\right]}_{\text{dielectric constant } \epsilon(\omega)} \vec{E}}$$

$$\boxed{\epsilon(\omega) = 1 + \frac{4\pi i}{\omega}\cdot\frac{\sigma_0}{1 - i\omega\tau}}$$

$$\epsilon(\omega) \simeq 1 + \frac{4\pi i}{\omega}\cdot\frac{\sigma_0}{-i\omega\tau} \boxed{= 1 - \frac{\omega_p^2}{\omega^2}}$$

$\omega\tau \gg 1$

$$\boxed{\omega_p^2 = \frac{4\pi n e^2}{m}}$$

plasma frequency

$$\left[-\nabla^2 \overset{!}{\cancel{+}} \epsilon(\omega)\left(\frac{\omega}{c}\right)^2\right] \vec{E}(\vec{r},t) = 0$$

$$\vec{E}(\vec{r},t) = \vec{E}_0 \cdot \exp(i\vec{k}\cdot\vec{r} - i\omega t)$$

$$\vec{k}^2 + \epsilon(\omega)\left(\frac{\omega}{c}\right)^2 = 0 \qquad |\vec{k}| = \sqrt{\epsilon(\omega)}\cdot\frac{\omega}{c}$$

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \begin{cases} < 0 & \omega < \omega_p \\ > 0 & \omega > \omega_p \end{cases}$$

① $\omega < \omega_p$ $\quad \mathrm{Im}|\vec{k}| \neq 0$ $\quad$ exponential decay with $\vec{r}$

non-propagating waves

② $\omega > \omega_p$ $\quad \mathrm{Im}|\vec{k}| = 0$ $\quad$ oscillations in $\vec{r}$-space

propagating waves

Is $\omega_p \tau \gg 1$? V: Transparency of alkali metals in UV

$$\boxed{\tau \sim 10^{-14} - 10^{-15}\ \mathrm{sec}} \qquad \boxed{\omega_p \tau \sim 40 - 4}$$

$$\omega_p^2 = \frac{4\pi n e^2}{m} \sim 16\times 10^{30}\ \mathrm{sec}^{-2} \qquad \boxed{\omega_p \sim 4\times 10^{15}\ \mathrm{sec}^{-1}}$$

$$n \sim 10^{22}\ \mathrm{cm}^{-3} \qquad e = 4.8\times 10^{-10}\ (\mathrm{CGS})$$

$$m = 9.1\times 10^{-28}\ \mathrm{gm}$$

22.9

Charge density $\equiv$ plasma oscillations

now self-induced charge density $\rho \neq 0$

- Equation of continuity

$$\vec{\nabla}\cdot\vec{j} + \frac{\partial \rho}{\partial t} = 0 \qquad \vec{\nabla}\cdot\vec{j}_\omega(\vec{r}) = i\omega\rho_\omega(\vec{r})$$

$$A(\vec{r},t) = A_\omega(\vec{r})\exp(-i\omega t)$$

- $\vec{\nabla}\cdot E_\omega(\vec{r}) = 4\pi\rho_\omega(\vec{r})$

- $\vec{j}_\omega(\vec{r}) = \sigma(\omega)\vec{E}_\omega(\vec{r}) \Rightarrow \vec{\nabla}\cdot\vec{j}_\omega = \vec{\nabla}\cdot(\sigma\vec{E}_\omega)$

$$4\pi\sigma(\omega)\rho_\omega = i\omega\rho_\omega$$

$$[4\pi\sigma(\omega) - i\omega]\rho_\omega = 0$$

$$\boxed{\begin{array}{l}\text{Non-trivial solution} \quad \rho_\omega \neq 0 \\ \text{when} \quad 1 + \dfrac{4\pi i\sigma(\omega)}{\omega} = 0\end{array}}$$

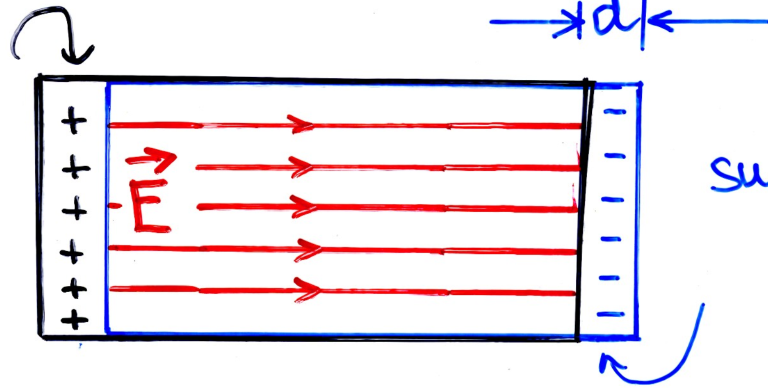
$$\omega^2 = \omega_p^2 = \frac{4\pi n e^2}{m}, \quad \omega\tau \gg 1$$

charge density wave $\equiv$ plasmon can propagate

22.10

positive surface charge density

$\sigma_+ = nde$



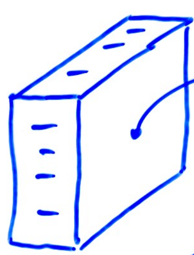
negative surface charge density

$\sigma_- = -nde$

the entire electron gas is displaced thru a distance d relative to fixed ions

Restoring force $\sim d \Rightarrow$ oscillations :

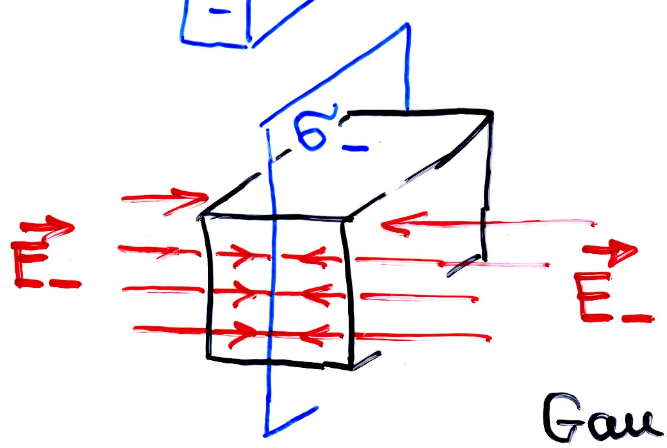
Uniform electric field $\vec{E}$



cross section A

$\Delta Q_- = -ned \cdot A$

$\sigma_- = \frac{\Delta Q_-}{A} = -ned$



$\vec{\nabla} \cdot E_- = 4\pi \rho_-$

$\int dV \, \vec{\nabla} \cdot \vec{E}_- = 4\pi Q_-$

Gauss theorem $\quad E_- = 2\pi \sigma_-$

$|\vec{E}| = |\vec{E}_+ + \vec{E}_-| = 4\pi nde$

22.11

Equation of motion for N electrons

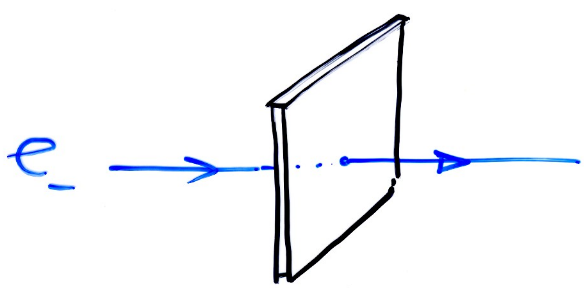
$$N m \ddot{d} = - Ne \cdot E$$

$$N m \ddot{d} = - Ne\, 4\pi n d e$$

oscillations $d = d_0 \cdot e^{-i\omega_p t}$ $\omega_p^2 = \frac{4\pi n e^2}{m}$

Plasmons as collective excitations of electron gas // many electrons involved //

Direct observations of plasmons



electrons loose energy in multiples of ω_p

thin metallic film

↓

excitations of several plasmons

22.12