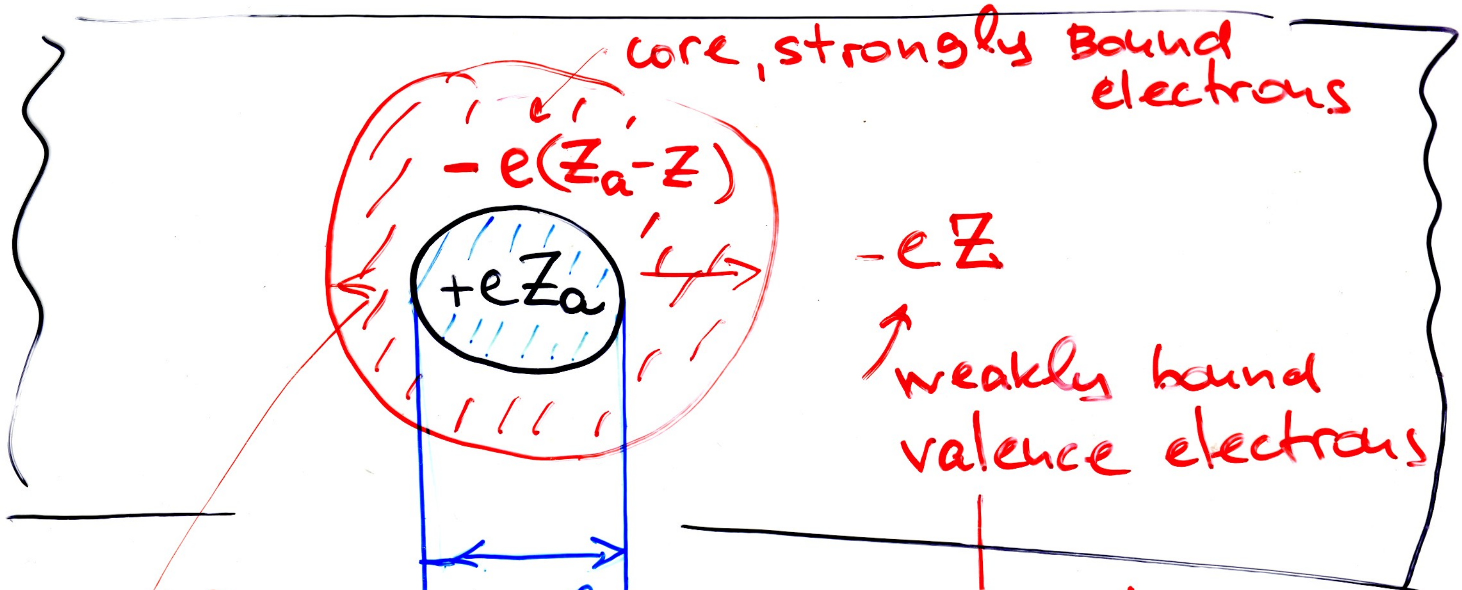
The Drude Theory of Metals

J. J. Thomson 1897 — discovery of the electron .... P. Drude 1900 — theory of metals

Classical kinetic theory of electron gas

Electrons : light mobile particles

Ions : heavy immobile particles $\equiv$ positive compensating background



nucleus 10^{-13} cm

conduction electrons

$$a_0 \sim \frac{\hbar^2}{m_e e^2}$$

$a_0 \sim 10^{-8}$ cm

Typical density of conduction electrons in metals

$n_e \sim 10^{22}\ \mathrm{cm}^{-3}$

	Z	$n_e\ (10^{22}\mathrm{cm}^{-3})$	r_s/a_0
Li	1	4.7	3.25
Cs	1	0.91	5.62
Be	2	24.7	1.87
Fe	2	17.0	2.12
Au	1	5.90	3.01
Ag	1	5.86	3.02
Cu	1	8.47	2.67
...			

Mean distance between electrons, r_s

3D: $\frac{4\pi}{3} r_s^3 \cdot n_e = 1 \Rightarrow r_s = \left(\frac{3}{4\pi n_e}\right)^{1/3}$

Dimensionless parameter r_s/a_0

$$2 < \frac{r_s}{a_0} < 3$$

Typically

$$a_0 = \frac{\hbar^2}{m_e e^2} = 0.5 \times 10^{-8}\ \mathrm{cm}$$

Bohr radius

Electrons = classical gas

Collisions: e-e interactions
e-ion interactions

Neglect e-e → "independent electron approximation"

Neglect e-ion → "free electron approx."

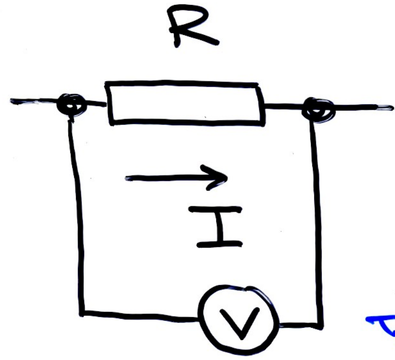
Phenomenological description of collisions: electron velocity is changed abruptly with a probability per unit time $1/\tau$

τ, the relaxation time

Probability that e experiences a collision during the time dt, P

$$P = \frac{dt}{\tau}$$

DC electrical conductivity



Ohm's Law

$V = I \cdot R$

Voltage drop — electric current — Resistance

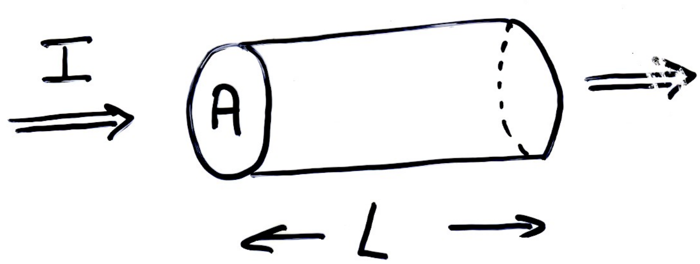
Characteristic of a <u>metal</u>,

resistivity ρ

$\vec{E}(\vec{r})$ local electric field

$\longrightarrow \vec{j}(\vec{r})$ local current density

$$\vec{j}(\vec{r}) = \vec{E}(\vec{r})/\rho$$



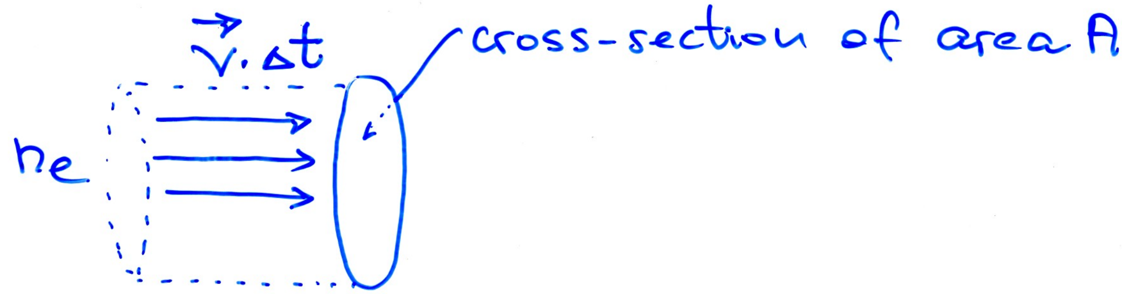
$$j = \frac{I}{A} \qquad V = E \cdot L = I \cdot R = \rho^{-1} E \cdot A \cdot R$$

$$R = \rho \cdot \frac{L}{A}$$

geometry dependent

Relate ρ with microscopic parameters

τ, m_e, n_e, e



current $I = \frac{\Delta Q}{\Delta t} = \frac{-e\cdot V\Delta t\cdot A\cdot n_e}{\Delta t} = -n_e e V\cdot A$

current density $\boxed{\vec{j} = -ne\vec{V}}$ $\quad n_e \to n$, $m_e \to m$ ...

local average electron velocity

$$\vec{V} = \overline{\vec{v}(t)} = \overline{\vec{v}_0 - \frac{e\vec{E}}{m}\cdot t} = -\frac{e\vec{E}}{m}\tau$$

$\vec{v}_0$ – velocity right after the collision; $\overline{\vec{v}_0} = 0$

$\overline{t}$ = mean time between collisions $= \tau$

$$\vec{j} = -ne\cdot\left(-\frac{e\vec{E}}{m}\tau\right) = \frac{ne^2\tau}{m}\cdot\vec{E} \equiv \sigma\vec{E} = \vec{E}/\rho$$

$$\boxed{\text{conductivity } \sigma = \frac{ne^2\tau}{m}} = \frac{1}{\rho}$$

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$$\tau = \boxed{\frac{m}{ne^2\rho}}\,?$$

easily measurable

resistivity

$\rho = \rho(T)$ increases in metals with T

$$\tau \sim 10^{-14} \div 10^{-15}\ \text{sec}$$

Mean free path $\ell = v_0 \cdot \tau$

average electronic speed

WRONG!

Drude: classical equipartition

$$\frac{1}{2} m v_0^2 = \frac{3}{2} k_B T \Rightarrow v_0 \approx 10^7 \frac{\text{cm}}{\text{sec}},\ T_{room}$$

$$\ell \approx 10 \div 1\ \text{Å} \sim \text{interatomic distance}$$

Quantum mechanics:

$$v_0 \sim v_{Fermi} \sim \frac{k_F}{m} \sim n^{1/3}$$

$$\frac{\frac{4\pi}{3} k_F^3}{(2\pi\hbar)^3} = n$$

two orders of magnitude larger!

$\Rightarrow$ $\ell \sim 1\text{cm}$ at low T

We can treat τ as a phenomenological parameter in a Drude theory

(Quantum mechanics hidden)

OK if classical theory applicable (?) to different phenomena

Our goals:

- $\sigma(B)$: when magnetic field $\vec{B}$ present
  uniform, static
  Hall effect

- $\sigma(\omega)$: when time-dependent electric field $\vec{E}\, e^{-i\omega t}$ present
  uniform, oscillating
  AC conductivity

- thermal conductivity of a metal

Forces $\vec{f}$ acting on e + collisions

$$\vec{j} = -\frac{ne}{m} \cdot \vec{p}(t)$$

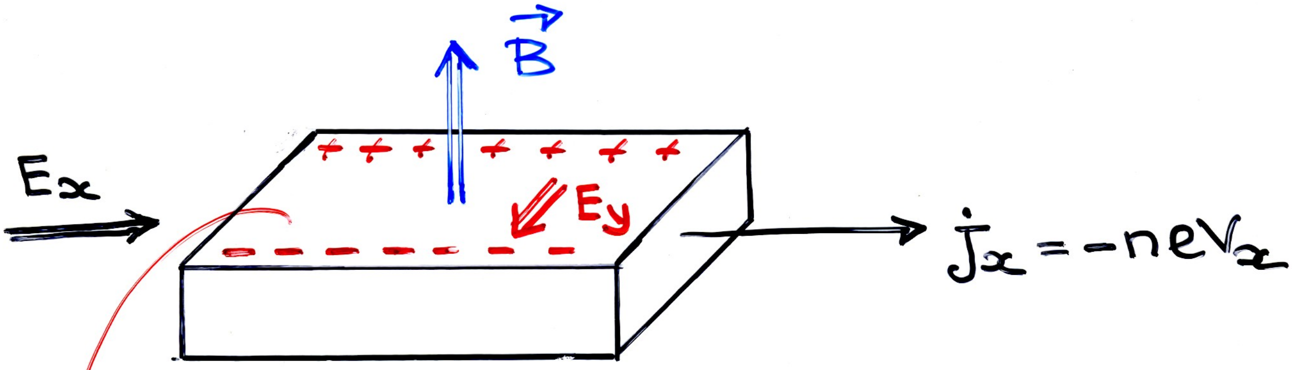
dynamics of momentum?

$$\boxed{\frac{d\vec{p}(t)}{dt} = -\frac{\vec{p}(t)}{\tau} + \vec{f}(t)}$$

Hall effect and magnetoresistance

moving e in $\vec{B}$: the Lorentz force

$$\vec{f} = -e\,\frac{\vec{v}}{c}\times\vec{B} \qquad \vec{f} \perp \vec{v}$$

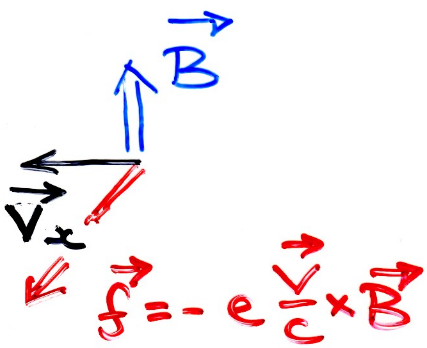


"separation" of charge

$\Rightarrow$ E_y appears to compensate $\vec{f}$

↓

then current flows in the x-direction



- Magnetoresistance

$$\rho_{xx} = \rho(B) = \frac{E_x}{j_x(B)}$$

- Hall coefficient (E.H. Hall 1879)

$$R_H = \frac{E_y}{j_x(B)\cdot B} = \frac{\rho_{yx}}{B}$$

↑ sign of the charge!

$\boxed{\text{Note: } R_H \text{ is often also used for } \rho_{yx}!}$

Simple calculations

$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{mc} \times \vec{B}\right) - \frac{\vec{p}}{\tau}$$

Steady-state regime: $\frac{d\vec{p}}{dt} = 0$

$$\begin{cases} -eE_x - \omega_c p_y - \frac{p_x}{\tau} = 0 \\ -eE_y + \omega_c p_x - \frac{p_y}{\tau} = 0 \end{cases}$$

$\omega_c = \frac{eB}{mc}$ cyclotron frequency

$\sigma_0 = \frac{ne^2\tau}{m}$ DC conductivity

$\vec{j} = -ne\frac{\vec{p}}{m}$ current density

$\times$ $-\frac{ne\tau}{m}$

$$\begin{cases} \sigma_0 E_x = \omega_c\tau j_y + j_x \\ \sigma_0 E_y = -\omega_c\tau j_x + j_y \end{cases}$$

No transverse current: $j_y = 0 \Rightarrow$ Hall field E_y

$$E_y = \frac{1}{\sigma_0}\cdot(-\omega_c\tau)\cdot j_x \equiv R_H \cdot B \cdot j_x$$

Hall coefficient $\boxed{R_H = -\frac{1}{nec}}$!!!

R_H depends $\boxed{\text{only}}$ on charge density

in this simple consideration

- classical theory
- τ- approximation

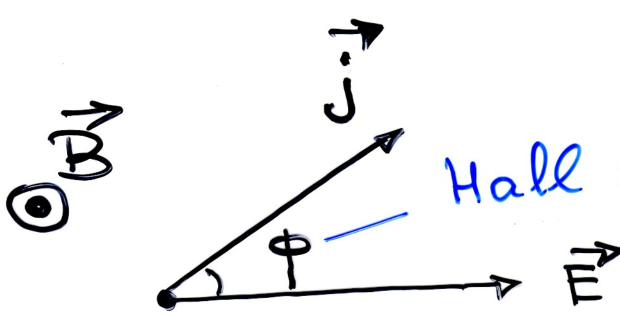
- no B- dependence
- no τ- dependence
- no T- dependence ...

More elaborate theory shows that

$$R_H = -\frac{1}{nec}$$ is a limiting case for some materials at <u>high B</u> $\quad \omega_c\tau \gg 1$

(one of) Measure of the strength of B

$\omega_c\tau \xleftarrow{①}$ how many revolutions in B e can make between collisions



Hall angle $\quad \boxed{\tan\phi = \omega_c\tau}$

$\omega_c t \ll 1$

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