Recipe for Phys 221

1. Study hard (you may ask other students)
2. Come to class
3. Do the homework (minimal requirements)
4. Do not stay behind
5. If you have questions, get them answered
6. If you have a problem, tell me about it
Chapter 1: Tooling Up

In this chapter we shall introduce the following concepts which will be used throughout this quarter (and beyond)

1. Units and systems of units
2. Conversion of units
3. Dimensional analysis
4. Order-of-magnitude estimates

I (not you!) skip: The building blocks of matter
In Phys221 we study mechanics that deals with the motion of physical bodies using Newton’s equations. These equations yield accurate results provided that:

1. The bodies in question are macroscopic (roughly speaking large, e.g. a car, a mouse, a fly)

2. The bodies do not move very fast. How fast? The yardstick is the speed of light in vacuum. 
   \[ c = 3 \times 10^8 \text{ m/s} \]
We must measure!

Example: I step on my bathroom scale and it reads 150
150 what? 150 lb? 150 kg?

For each measurement we need units.

Do we have to define arbitrarily units for each and every
physical parameter?

The answer is no. We need only define arbitrarily units for the following four parameters:

Length, Mass, Time, Electric Current

In Phys221 we will need only units for the first three. We define the units for electric current in Phys222
In this course we shall use the **SI** (systeme internationale) system of units as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (L)</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>Mass (M)</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time (T)</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Electric Current</td>
<td>Ampere</td>
<td>A</td>
</tr>
</tbody>
</table>

All other units follow from the arbitrarily defined four units listed above.

**Note:** SI used to be called the “MKSA” system of units.

**Note:** In CGS the first three units are enough.
Basic Quantities and Their Dimension

+ Standards of
  Mass  M
  Length  L
  Time  T
The meter

1 meter $\equiv \frac{AB}{10^7}$

A particular longitudinal line AB?
The standard meter

It is a bar of Platinum-Iridium kept at a constant temperature. The meter is defined as the distance between the two scratch marks.

Redefined in 1983: the distance traveled by light in vacuum during the Time of 1/299,792,458 sec.
The kilogram (kg)

It is defined as the mass equal to the mass of a cylinder made of platinum-iridium made by the International Bureau of Weights and Measures. All other standards are made as copies of this cylinder. Established in 1887, still going strong…
The second (s)

The second is defined as the duration of the mean solar day divided by 86400.

The mean solar day is the average time it takes the earth to complete one revolution around its axis.

Where does the 86400 come from?

1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds
Thus: 1 day $24 \times 60 \times 60 = 86400$ s

Redefined in 1960: now “atomic clock” in use
Other units

Such as millimeter (mm) and nanosecond (ns), where prefix denote multiplier of the basic units based on various powers of ten:

1 mm = 10^{-3} m

1 km = 10^{3} m

1 ns = 10^{-9} s
Conversion of units

1 m = 39.37 in = 3.281 ft

1 in = 0.0254 m = 2.54 cm

1 mi = 1609 m = 1.609 km

etc.
Dimensional Analysis

The dimensional analysis of a physical parameter such as velocity \( v \), acceleration \( a \), etc expresses the parameter as an algebraic combination of length \([L]\), mass \([M]\), and time \([T]\). This is because all measurements in mechanics can be ultimately be reduced to the measurement of length, mass, and time.

L, M, and T are known as primary dimensions.

How do we derive the dimensions of a parameter? We use an equation that involves the particular parameter we are interested in. For example: \( v = x/t \). In every equation [Left Hand Side] = [Right Hand Side]

Thus: \([v] = L/T = L \cdot T^{-1}\)
Note: The dimensions of a parameter such as velocity does not depend on the units. \([v] = L \ T^{-1}\) whether \(v\) is expressed in m/s, cm/s, or miles/hour.

Dimensional analysis can be used to detect errors in equations.

Example:

\[ h = gt^2/2 \]  

\([\text{LHS}] = [\text{RHS}]\)

\([\text{LHS}] = [h] = L\)

\([\text{RHS}] = [gt^2/2] = [g][t^2] = LT^{-2} \ T^2 = L\)

Indeed \([\text{LHS}] = [\text{RHS}] = L\) as might be expected from the equation \(h = gt^2/2\) which we know to be true.
**Note 1:** If an equation is found to be dimensionally incorrect then it is incorrect

**Note 2:** If an equation is dimensionally correct it does not necessarily mean that the equation is correct.

**Example:** Let's try the (incorrect) equation

\[ h = gt^{2/3} \]

[LHS] = L

[RHS] = [g][t^2] = L

Even though [LHS] = [RHS] equation \( h = gt^{2/3} \) is wrong!
Dimensional Analysis, cont

- Cannot give numerical factors: this is its limitation
- Dimensions of some common quantities:

Table 1.6

<table>
<thead>
<tr>
<th>System</th>
<th>Area (L²)</th>
<th>Volume (L³)</th>
<th>Speed (L/T)</th>
<th>Acceleration (L/T²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>m²</td>
<td>m³</td>
<td>m/s</td>
<td>m/s²</td>
</tr>
<tr>
<td>U.S. customary</td>
<td>ft²</td>
<td>ft³</td>
<td>ft/s</td>
<td>ft/s²</td>
</tr>
</tbody>
</table>
Estimates and Order-of-Magnitude Calculations

- Approximation based on a number of assumptions
  - May need to modify assumptions if more precise results are needed

- An order of magnitude of a certain quantity is the power of 10 of the number that describes that quantity

- The symbol $\sim$ for “is on the order of”
Estimates

\[ N_A = \# \text{ of particles in 1 mole} = 6.022 \times 10^{23} \]

Or \(10^{24}\)?

\[ N_A \sim 10^{23} \quad \text{Avogadro’s number} \]

Order-of-magnitude calculations

Breaths in a Lifetime?

Lifetime = 70 y \sim 70 \times 400 \text{ days} \times 25 \text{ h/day} \times 60 \text{ min/h} \sim \]
\sim (70 \times 60) \times (400 \times 25) \text{ min} \sim (4 \times 10^3) \times 10^4 \sim 4 \times 10^7 \text{ min}

Breaths \sim 4 \times 10^7 \text{ min} \times 10 \text{ breath/min} \sim 4 \times 10^8
Uncertainty in Measurements

- There are uncertainty in every measurement
- This uncertainty carries over through the calculations
  - need a technique to account for this uncertainty!

- We will use rules for significant figures to approximate the uncertainty in results of calculations
Significant figures

The measured values are known only to within the limits of experimental uncertainty.

1 500 g  ?  possibility of misinterpretation

Use scientific notations to indicate # of significant figures!

1.5 \times 10^3 \text{ g}  \quad \text{two significant figures}

1.500 \times 10^3 \text{ g}  \quad \text{four significant figures}
Uncertainty in Measurement

There is no such thing as a perfectly accurate measurement. Each and every measurement has an uncertainty due to: 1. the observer, 2. the instrument, and 3. the procedure used.

How do we express the uncertainty in a measurement?

Assume that we are asked to measure the length \( L \) of an object with the ruler shown on the previous page. The smallest division on this ruler is 1 mm. The uncertainty \( \delta L \) in \( L \) using that particular ruler is 1 mm. (If one is careful one can reduce it to 0.5 mm). If \( L \) is found to be 21.6 cm we write this as:

\[
L = (21.6 \pm 0.1) \text{ cm}
\]

This simply means that the real value is somewhere between 21.5 cm and 21.7 cm. We can give \( L \) using three significant figures.

Percentage Uncertainty = \( \frac{\delta L}{L} \times 100 = \frac{0.1}{21.6} \times 100 = 0.5\% \)
We are given the same ruler and are asked to measure the width $L_1$ and height $L_2$ of the rectangle shown.

$L_1 = (21.6 \pm 0.1) \text{ cm}$

$L_2 = (27.9 \pm 0.1) \text{ cm}$

Area $A = L_1L_2 = 21.6 \times 27.9 \text{ cm}^2 = 602.6 \text{ cm}^2$

The uncertainties $\delta L_1$ and $\delta L_2$ in the measurement of $L_1$ and $L_2$ will result in an error $\delta A$ in the calculated value of the rectangle area $A$. This is known as error propagation.

Step 1: 

$\frac{\delta L_1}{L_1} = \frac{0.1}{21.6} = 0.005$ 

$\frac{\delta L_2}{L_2} = \frac{0.1}{27.9} = 0.004$

Step 2: 

$\frac{\delta A}{A} = \frac{\delta L_1}{L_1} + \frac{\delta L_2}{L_2} = 0.005 + 0.004 = 0.009$

$\delta A = 0.009A = 0.009 \times 602.6 = 5 \text{ cm}^2$

$A = (603 \pm 5) \text{ cm}^2$
Significant Figures

- A significant figure is one that is **reliably** known
- All non-zero digits are significant
- Zeros are significant
  - when between other non-zero digits
  - after the decimal point and after another significant figure
  - can be clarified by using scientific notation
Quick Quiz

How many significant figures are in each of the following numbers?

a) 1.234
b) 1.2340
c) 1.234 \times 10^{-3}
d) 1.2340 \times 10^{-3}
e) 1234
f) 12340
g) 0.012340

Answer:

4
5
4
5
4
4 or 5
5 (or 6?)
Operations with Significant Figures: General Principle

When combining measurements with different degrees of accuracy and precision: the accuracy of the final answer can be no greater than the least accurate measurement.

\[
\begin{align*}
150.0 \text{ g H}_2\text{O} + 0.507 \text{ g salt} \\
\text{The cost of the copper in an old penny} \\
\text{Mass} = 2.531 \text{ grams} \\
\text{The price of copper is 67 cents per pound} \\
2.531 \text{ g} \times (1 \text{ lb}/453.6 \text{ g}) \times (67 \text{ cents/lb})
\end{align*}
\]
Addition and Subtraction

When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.

150.0 g H₂O
+ 0.507 g salt
150.5 g solution

http://chemed.chem.purdue.edu/genchem/topicreview/bp/ch1/sigfigs.html#determ
Multiplication and Division

The cost of the copper in an old penny?
Mass of one penny = 2.531 grams, pure Cu
The price of copper is 67 cents per pound

\[ 2.531 \text{ g} \times \left( \frac{1 \text{ lb}}{453.6 \text{ g}} \right) \times (67 \text{ cents/lb}) \]
\[ = 0.373847 \text{ cents} \]

Your calculator gives probably this value. Do you believe in this?

How many significant figures?

Only 2!

Round off!

Physically correct answer: \( 0.37 \text{ cents} \)
Operations with Significant Figures

- multiplying or dividing: the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures

- adding or subtracting: the number of decimal places should equal the smallest number of decimal places of any term in the sum