MATHEMATICS DEPARTMENT SEMINAR

Critical Points and the Schwarzian Derivative

Dr. Brian Ryals

California State University Bakersfield

2:00 PM TO 3:00 PM

Wednesday, October 21, 2026

Science III, Room 123

Abstract

Many discrete population models describe change from one generation to the next by repeatedly applying a function. Even simple functions can produce attracting equilibria, periodic cycles, and more complicated behavior. What can calculus tell us about these iterates? One useful tool is the Schwarzian derivative, an expression involving the first three derivatives of a function. Its sign places restrictions on attracting periodic orbits. For rational functions, we ask whether this sign can be determined from simpler information: the locations of zeros, poles, and critical points. We will show that a real rational map of degree at least two has negative Schwarzian at every regular real point whenever all of its critical points, including those in the complex plane and at infinity, are real. The polynomial case has an elementary calculus proof; the rational case brings in a classical theorem from complex dynamics. We will also discuss a complete positive, negative, and mixed sign classification in degree three, and examples showing what changes in higher degree. Familiarity with calculus and complex numbers will be sufficient to understand most of the talk.

STUDENTS ARE WELCOME!