CALIFORNIA STATE UNIVERSITY, BAKERSFIELD MATHEMATICS FIELD DAY
2026

Individual Medley, Varsity Level

There are 25 problems. You will have 50 minutes. You will only have to turn in the answer sheet – you may keep the test. Each correct answer is worth four points. Each incorrect answer will receive a one point penalty. If you believe there is an error of some sort in a particular problem, please quietly discuss it with one of the proctors. For each of the questions, fill in the appropriate circle on the answer sheet. When the exam is over, give your answer sheet to the proctor.

Calculators or cellphones of any kind are not allowed.

GOOD LUCK!

1. Suppose r and s are the roots of $x^2 - 2026x + 1 = 0$. What is rs?

   (A) 1 (B) 2026 (C) $\sqrt{1518}$ (D) 1518 (E) None of these

2. Determine the number of divisors of 20^{26}.

   (A) 1352 (B) 1431 (C) 2026 (D) 6^{26} (E) None of these

3. A circle has radius 10. A chord has length 12. What is the distance from the center of the circle to the chord?

   (A) 6 (B) 7 (C) 8 (D) 9 (E) None of these

4. Consider the fraction
$$\frac{1+3+5+\cdots+2021+2023+2025}{2+4+6+\cdots+2022+2024+2026},$$
   that is the sum of the odd positive integers up to 2026 divided by the sum of even positive integers up to 2026. What does this fraction reduce to?

   (A) $\frac{1013}{1014}$ (B) $\frac{4053}{4054}$ (C) $\frac{7}{8}$ (D) $\frac{2025}{2026}$ (E) None of these

5. What is the smallest integer that is larger than $\left(\sqrt{2}+1\right)^3$?

   (A) 12 (B) 13 (C) 14 (D) 15 (E) None of these

6. Suppose we wrote $\frac{1}{7}$ as a decimal. What is the 2026-th digit after the decimal point?

   (A) 2 (B) 5 (C) 7 (D) 8 (E) None of these

7. How many real values of x satisfy $2026^x = 2026 - x$?

(A) 0 (B) 1 (C) 2 (D) 2026 (E) None of these

8. How many distinct arrangements of the letters in "PARALLEL" have all three L's consecutive?

(A) 120 (B) 180 (C) 240 (D) 360 (E) 720

9. A triangular number T_n is of the form $T_n = \dfrac{n(n+1)}{2}$ for some positive integer n. The first several triangular numbers are $T_1 = 1$, $T_2 = 3$, $T_3 = 6$, $T_4 = 10$, $T_5 = 15$, $T_6 = 21, \cdots$. Sometimes adding a triangular number to another triangular number still gives a triangular number, for instance $T_3 + T_5 = T_6$. In fact, $T_{20} + T_{26}$ is also a triangular number. Which triangular number is it?

(A) T_{25} (B) T_{30} (C) T_{33} (D) T_{46} (E) T_{67}

10. Which of the following is equivalent to $\sqrt{45 + \sqrt{2000}} + \sqrt{45 - \sqrt{2000}}$?

(A) 10 (B) $6\sqrt{3}$ (C) $5\sqrt{2}$ (D) 12 (E) None of these

11. Two fair dice are rolled repeatedly until the sum is 7. What is the probability it takes exactly 3 rolls?

(A) $\frac{5}{36}$ (B) $\frac{25}{216}$ (C) $\frac{1}{36}$ (D) $\frac{125}{216}$ (E) None of these

12. In $\triangle ABC$, $AB = 13$, $AC = 14$, and $BC = 15$. What is the area of the triangle?

(A) 72 (B) 78 (C) 84 (D) 90 (E) None of these

13. Suppose you wrote all integers from 1 to 2026 on your paper. You erased exactly one number, and the sum of the remaining numbers is divisible by 2026. Which number was erased?

(A) 26 (B) 128 (C) 1013 (D) 2026 (E) None of these

14. A function satisfies $f(1) = 2026$ and $\dfrac{f(n)}{n-1} = \dfrac{f(n-1)}{n} + \dfrac{2026}{n^2 - n}$ for $n \geq 2$. Find $2026 \cdot f(2026)$.

(A) 2026 (B) $2025 \cdot 2026$ (C) 2026^2 (D) $2026 + 2026^2$ (E) None of these

15. How many positive solutions x are there to $\frac{(x!)^2}{(x+1)!(x-1)!} = \frac{2025}{2026}$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of these

16. What is the smallest positive integer whose digits sum to 2026?

(A) $10^{225} - 1$ (B) $2 \cdot 10^{225} - 1$ (C) $10^{226} - 1$ (D) $2 \cdot 10^{226} - 1$ (E) None of these

17. If $x^2 + \dfrac{1}{x^2} = 18$, which of the following could be the value of $x^3 - \dfrac{1}{x^3}$?

(A) -19 (B) 38 (C) -57 (D) 76 (E) None of these

18. What is the last digit of $7^{6^{5^{4^{3^{2^1}}}}}$?

(A) 1 (B) 3 (C) 5 (D) 9 (E) None of these

19. If $\sin(\theta) + \sin^2(\theta) = 1$, what is the value of $\cos^2(\theta) + \cos^4(\theta)$?

(A) $\dfrac{1}{4}$ (B) 1 (C) $\dfrac{\sqrt{5}-1}{2}$ (D) $\dfrac{3-\sqrt{5}}{2}$ (E) None of these

20. Let $g(n)$ be the sum of the square of the digits of n. For instance, $g(2026) = 2^2 + 0^2 + 2^2 + 6^2 = 44$. If we take this number and plug it into g again, we obtain $g(44) = 32$. If we keep substituting, we obtain $g(32) = 13$, $g(13) = 10$, and $g(10) = 1$. What is the next positive integer n larger than 2026 with the property that if we keep substituting the result into g, we eventually obtain 1?

(A) 2036 (B) 2039 (C) 2062 (D) 2063 (E) None of these

21. A polynomial f of degree 2026 has $f(-1) = \dfrac{1}{2}f(0)$, $f(0) = \dfrac{1}{2}f(1)$, and $f(1) = 2026$. What is the sum of the coefficients of the even degree terms of f?

(A) $\dfrac{1}{4} \cdot 2026$ (B) $\dfrac{3}{4} \cdot 2026$ (C) $\dfrac{5}{8} \cdot 2026$ (D) $\dfrac{9}{16} \cdot 2026$ (E) None of these

22. Considered the circle of radius 2 centered at $(0, 0)$. Inside the circle, there is a point P with coordinates $(1, 1)$. A line through P meets the circle at two points: A and B. Which expression is independent of how the line was drawn?

(A) $PA + PB$ (B) $PA - PB$ (C) $PA \cdot PB$ (D) PA/PB (E) None of these.

23. The positive integers are concatenated into a single string $123456789101112131415161718\cdots$. What digit is in the 2026th spot?

(A) 1 (B) 2 (C) 3 (D) 7 (E) None of these.

24. Define the super factorial of a positive integer n as the product of the first n factorials. For example, the super factorial of 5 is $1! \cdot 2! \cdot 3! \cdot 4! \cdot 5! = 34560$, which has 1 zero at the end. How many zeros are at the end of the super factorial of 26?

(A) 6 (B) 26 (C) 60 (D) 62 (E) None of these

25. How many integers n with $1 \leq n \leq 2026$ have $n^2 + n$ divisible by 12?

(A) 672 (B) 673 (C) 674 (D) 675 (E) 676