(1) Three coins are tossed: two nickels and a dime. We pay 10 cents to enter the game and receive those coins which fall heads up. What is the probability of winning some money?
   (hint: write the sample space!)

(2) In a lottery, 6 numbers are drawn out of 49. Find the probability that:
   (a) 1,3,5,7,9 and 11 are drawn.
   (b) 44 is one of the numbers drawn.

(3) From a deck of 52 cards, we draw a sample without replacement. What is the probability that an Ace will appear in the fifth draw?

(4) A deck of 52 cards contains 13 hearts. Suppose that the cards are shuffled and distributed among four players A, B, C, and D so that each player receives 13 cards. Determine the probability that player A will receive 6 hearts, player B will receive 4 hearts, player C will receive 2 hearts and player D will receive 1 heart.

(5) A deck of 52 cards contains 12 picture cards (4 Kings, 4 Queens and 4 Jacks). If the 52 cards distributed in a random manner among four players in such a way that each player receives 13 cards, what is the probability that each player will receive three picture cards?

(6) How likely is it that the word ABRACADABRA will show if the letters A, A, A, A, A, B, B, C, D, R, R are shuffled randomly?

(7) If boys are born with probability 0.6 and girls are born with probability 0.4, what is the probability that in a family with three children, exactly one is a girl?
(8) **Bonus Problem 1** Prove the following two equalities:

(a) For every three events $A_1$, $A_2$ and $A_3$ we have

$$
Pr(A_1 \cup A_2 \cup A_3) = Pr(A_1) + Pr(A_2) + Pr(A_3) - \left[ Pr(A_1 \cap A_2) + Pr(A_1 \cap A_3) + Pr(A_2 \cap A_3) \right] + Pr(A_1 + A_2 + A_3)
$$

**Hint** Remember that we proved:

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

(b) Prove that for every $n$ events $A_1, A_2, ..., A_n$,

$$Pr\left( \bigcup_{i=1}^{n} A_i \right) = \sum_{i=1}^{n} Pr(A_i) - \sum_{i<j} Pr(A_i \cap A_j) + \sum_{i<j<k} Pr(A_i \cap A_j \cap A_k) - \sum_{i<j<k<l} Pr(A_i \cap A_j \cap A_k \cap A_l) + ... + (-1)^{n+1} Pr(A_1 \cap A_2 ... \cap A_n)$$

**Hint** Use the previous statements and **mathematical induction**.

(9) **Bonus Problem 2** We draw cards, one at a time, at random and successively from an ordinary deck of 52 cards with replacement. What is the probability that an ace appears before a face card?

(10) Suppose that $P(A) \neq 0$ and $P(B) \neq 0$. Show that the following conditions are equivalent:

(a) $P(A) = P(A|B)$,

(b) $P(B) = P(B|A)$,

(c) $P(A)P(B) = P(A \cap B)$.

(11) You have three coins in your pocket, two fair ones but the third biased with probability of heads $p$ and tails $1 - p$. One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins?