Counting and Combinatorial Methods (2)

(E1) In how many ways can two students and seven magicians be selected from ten students and ten magicians? (assume that no student is a magician!)

(E2) In a group of 45 mathematicians 32 are trying to prove a particular theorem. If three of the mentioned mathematicians are Teddy, Steve and Bill, what is the probability that all three are working on the same proof?

(E3) In a group of 25 senators, 11 are against military action, eight are for military action, and the rest are indifferent. A random sample of size six is selected from the 25 senators. what is the probability that: (a) all of them are in favor military action? (b) all six selected ones have same opinion regarding military action?

(E4) In Maryland’s lottery, players pick six different integers between 1 and 49, order of selection is irrelevant. Six numbers among 49 are randomly selected as winning numbers. A player hits the jackpot if she/he matches all six numbers. The second big prize is rewarded to person(s) matching five numbers and the third prize goes to person(s) matching 4. Find the probabilities that: (a) Sam’s ticket wins the jackpot. (b) Sam’s ticket wins the second prize. (c) Sam’s ticket wins the third prize.

(E5) From an ordinary deck of 52 cards, seven are drawn at random and without replacement. What is the probability that at least one of the cards is a king?

(E6*) What is the probability that a poker hand is a full house? In the game of poker, a hand of 5 randomly selected cards is called full if there are three cards are from one denomination and the other 2 cards are from another denomination. For example, a hand of three kings and two 3s.

(E7) The mathematics department consists of 25 full professors, 15 associate professors, and 35 assistant professors. A committee of 6 is selected at random from each faculty of the department. (a) Find the probability that all members of the committee are assistant professors. (b) What is the probability that the committee of 6 is composed of 2 full professors, 3 associate professors and 1 assistant professor?

(E8) In a hand of 13 cards chosen from an ordinary deck of 52, find the probability that the hand is composed of exactly 3 clubs, 4 diamonds, 4 hearts and 2 spades.
(E9*) **matching problem** An absentminded professor wrote $n$ letters and sealed them in envelopes before writing the addresses on the envelopes. Then he wrote $n$ addresses on the envelopes at random. What is the probability that at least one letter was addressed correctly?

**Hint** The Taylor expansion of $e^x$ might be used somewhere in order to give an approximate answer.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

if $n \to \infty$