Problem 10

Since $t$-value $= 1.12$ and $df = 29$ then the closest right tail probabilities from the table are 0.10 and 1.15. However, this is a two-sided test hence: $0.20 < p\text{-value} < 0.30$ therefore we can not reject at any of 0.05 or 0.10 levels.

Problem 11

The hypotheses are:

$H_0 : \mu_{\text{stats}} = \mu_{\text{stats+exercise}}$

$H_a : \mu_{\text{stats}} < \mu_{\text{stats+exercise}}$

with a p-value as large as 0.87 we can NOT possibly reject the null hypothesis. That is, there is not enough evidence provided by data against the null hypothesis. In general, relying solely on these results misses the fact that students might be coming from various backgrounds, their preliminary familiarity with statistics is not known, their basic desire in doing exercise is ignored, etc. Generally speaking, we need to learn about the “lurking variables” as much as possible. Also, the sample size, the distribution of the population, etc are not reported.

Problem 12

The message here is that with keeping the sample size fixed, but with increasing the confidence level we are going to make the confidence intervals wider because we are making the margin of error or $t_{\text{critical}} \times \frac{s}{\sqrt{n}}$ larger.

```r
> crop<-c(138,139.1,113,132.5,140.7,109.7,
118.9,134.8,109.6,127.3,115.6,130.4,130.2,111.7,105.5)

> t.test(crop,alternative="two.sided",conf.level=.90)

One Sample t-test

data:  crop
t = 39.1077, df = 14, p-value = 1.063e-15
alternative hypothesis: true mean is not equal to 0
90 percent confidence interval:
  118.2244 129.3756
sample estimates:
mean of x
  123.8

> t.test(crop,alternative="two.sided",conf.level=.95)
```
One Sample t-test

data:  crop
t = 39.1077, df = 14, p-value = 1.063e-15
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
   117.0104 130.5896
sample estimates:
mean of x
   123.8

> t.test(crop,alternative="two.sided",conf.level=.99)

One Sample t-test

data:  crop
t = 39.1077, df = 14, p-value = 1.063e-15
alternative hypothesis: true mean is not equal to 0
99 percent confidence interval:
   114.3765 133.2235
sample estimates:
mean of x
   123.8

Problem 13

The most important message in this problem is that by increasing the sample size rather radically, we are going to decrease the margin of error or $t_{critical} \times \frac{s}{\sqrt{n}}$ significantly. This phenomenon leads to an automatic rejection of the null hypothesis and it is yet another peculiar feature of the classical statistics (in particular, see answers to parts c and d of this question.) $H_0 : \mu = 475$

$H_a : \mu > 475$

(a) $t_{obs} = \frac{478 - 475}{(100/sqrt(1000))} = 0.30$ which yields a large p-value (about 0.40) and leads us to not rejecting the null hypothesis.

(b) $t_{obs} = \frac{478 - 475}{(100/sqrt(10000))} = 0.948$ with p-value of about 0.17 which provides more evidence against the null hypothesis but still not strong enough to reject it.

(c) $t_{obs} = \frac{478 - 475}{(100/sqrt(1000000))} = 3$ or strong evidence against the null hypothesis!

(d) $475.636 < \mu < 480.364$
Problem 14

(a) Let $\mu_{\text{diff}} = \mu_{\text{post}} - \mu_{\text{pre}}$
then:
$H_0 : \mu_{\text{diff}} = 0$
$H_a : \mu_{\text{diff}} > 0$

(b) The box plot does not reveal any outliers. In general though, the distribution is slightly left skewed.

(c) $t_{\text{obs}} = 2.0244$ with $p$–value $= 0.0286$. Therefore, we can NOT reject the null hypothesis at 0.01 level but we will reject it at 0.05!

(d) the confidence interval is $(0.2114 < \mu_{\text{diff}} < 0.2688)$

Finally, please note that I also conducted the same test using R by treating the sample of differences independently. This yielded in the same results as for the matched-pairs approach.

```r
> length(pre)
[1] 20
> post<-c(29,30,32,30,16,25,31,18,33,25,32,28,34,32,32,27,28,29,32,32)
> length(post)
[1] 20
> diff<-post-pre

> t.test(post,pre,paired=T)

    Paired t-test

data: post and pre
    t = 2.0244, df = 19, p-value = 0.05722
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
    -0.04914638  2.94914638
sample estimates:
    mean of the differences
                     1.45

> t.test(diff)

    One Sample t-test

data: diff
    t = 2.0244, df = 19, p-value = 0.05722
alternative hypothesis: true mean is not equal to 0
```
95 percent confidence interval:
\[-0.04914638 \text{ to } 2.94914638\]
sample estimates:
mean of x
1.45

> t.test(post, pre, paired = T, conf.level = .90)

Paired t-test
data: post and pre
t = 2.0244, df = 19, p-value = 0.05722
alternative hypothesis: true difference in means is not equal to 0
90 percent confidence interval:
0.2114938 \text{ to } 2.6885062
sample estimates:
mean of the differences
1.45

Problem 15

Here, the population proportion is 0.55. We have \( n = 450 \) for the sample size.

(a) Based on the central limit theorem, we have:
\[
\hat{p} \sim N(p, \sqrt{\frac{p(1-p)}{n}}).
\]

(b) We can write: \( Pr(0.54 < \hat{p} < 0.62) = Pr\left(\frac{0.54 - 0.55}{\sqrt{(0.55)(0.45)/450}} < z < \frac{0.62 - 0.55}{\sqrt{(0.55)(0.45)/450}}\right) \)
which is going to be calculated using the z-table or using R.

(c) More than half unlisted means less than half are listed. In other words, we want to find: \( Pr(\hat{p} < 0.5) \). We calculate this probability using the standardization technique explained for the previous part.

Problem 16

(a) \( 413.5577 < \mu_1 - \mu_2 < 634.7023 \). Since 0 is not covered by the interval, we can conclude that the population mean income of males is significantly larger than the females.

(b) We have large sample sizes for both populations guaranteeing the results of the \( t \)-test. We talked about the robustness of \( t \)-tests in class.

(c) The results here are valid only if the sample observations are \( iid \). This in a large sense guaranteed when SRS samples are obtained. Any systematic tendency in drawing samples might violate the validity of our results.
(d) Here is yet another example in which there is a potential for the existence of "lurking" or "extraneous" variables that are not counted for. The socio-economical status of the students families, their academic achievements, even their general behavior could play important roles in this problem.

**Problem 17**

The steps are included below. Particularly, note the use of the `var.test` command which performs an $F$-test for this problem. See section 4.4 at Dalgaard (page 89).

```r
> calcium<-c(7,-4,18,17,-3,1,10,11,-2)
> placebo<-c(-1,12,-1,-3,3,5,5,-2,-11,-1,-3)

> t.test(calcium,placebo,alternative="greater")

Welch Two Sample t-test

data:  calcium and placebo
t = 1.4378, df = 15.591, p-value = 0.08513
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 -1.022133     Inf
sample estimates:
mean of x  mean of y
 5.0000000  0.2727273

> 1-pt(1.4378,10)
[1] 0.09051913

> var(calcium)/var(placebo)
[1] 2.195532

> var.test(calcium,placebo,alternative="greater")

F test to compare two variances

data:  calcium and placebo
F = 2.1955, num df = 9, denom df = 10, p-value = 0.1182
alternative hypothesis: true ratio of variances is greater than 1
95 percent confidence interval:
 0.7269053     Inf
sample estimates:
 ratio of variances
    2.195532
```