CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2011
Individual Medley, Junior - Senior Level

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. If you finish early, you should double check your answers. When time is up and the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.
1. The roots of the quadratic \( a - 2bx + cx^2 \) are:
   
   a) \( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)
   
   b) \( \frac{b \pm \sqrt{(b/c)^2 - a/c}}{c} \)
   
   c) \( \frac{2b \pm \sqrt{4b^2 - 4ac}}{2c} \)
   
   d) \( \frac{-2b \pm \sqrt{4b^2 - 4ac}}{2c} \)
   
   e) \( \frac{b \pm \sqrt{b^2 - 4ac}}{2c} \)

2. Segments AB and CD do not intersect. But segments AD and BC intersect at E. Suppose \( \angle BAD - \angle BCD = x \). Which of the following equals \( \angle ABC - \angle ADC \).
   
   a) \( x \)
   
   b) \( 2x \)
   
   c) \( -x \)
   
   d) \( 180 - x \)
   
   e) Not enough information is given

3. A regular pentagon is inscribed in a circle. Through the center of the circle are drawn lines parallel to each of the sides of the pentagon. All together, the circle is now divided into how many regions?
   
   a) \( 10 \)
   
   b) \( 15 \)
   
   c) \( 20 \)
   
   d) \( 25 \)
   
   e) \( 35 \)

4. Mary, Joe, and David each roll a standard die once. What is the probability that the three rolls all result in different numbers? Round your answer to the nearest percent.
   
   a) \( 50 \)
   
   b) \( 52 \)
   
   c) \( 55 \)
   
   d) \( 56 \)
   
   e) \( 67 \)

5. At step 0, we have an equilateral triangle with side length 1. To get from one step to the next, to each edge on the perimeter is added another equilateral triangle with side length 1 (except that sometimes, two exterior sides will determine the same triangle – in this case only one new edge is added). What is the perimeter of the figure at step 4?
   
   a) \( 15 \)
   
   b) \( 18 \)
   
   c) \( 21 \)
   
   d) \( 24 \)
   
   e) \( 27 \)
6. A fourth degree polynomial has leading coefficient 1 and all other coefficients are real numbers. Two of the roots are $1-2i$ and $3+i$. What is the sum of the $x^3$ coefficient and the constant term?
   a) 50  
   b) 42  
   c) 45  
   d) 58  
   e) none of these

7. A price of a dress was marked up by 30% and then, a month later it was marked up again by another 30%. A year later it still had not sold. The shopkeeper marked it back down to the original price. This represented a discount of what percent of the latest price. Answer to the nearest whole percent.
   a) 40  
   b) 41  
   c) 52  
   d) 59  
   e) 69

8. Suppose $x^2 + bx + c$ has roots $r$ and $s$. Which quadratic has roots $2r$ and $3s$?
   a) $x^2 + 6bx + 5c$  
   b) $x^2 + 2bx + 6c$  
   c) $x^2 + (2b+s)x + 6c$  
   d) $x^2 + (2b-s)x + 6c$  
   e) $x^2 + (2r+3s)x + 6c$

9. Suppose $y$ is a real number larger than 1. In the interval $[0, 2\pi]$, how many values of $\theta$ are there such that
   \[ \frac{\sin 2\theta + \cos 2\theta}{\sin 2\theta - \cos 2\theta} = y \]?
   a) 0  
   b) 1  
   c) 2  
   d) 3  
   e) 4

10. To make 12 widgets, it takes 8 workers 16 days, when they work 6 hours each day. How many days would it take to make 30 widgets, if 12 workers were available and they each worked 8 hours per day?
   a) 5  
   b) 10  
   c) 15  
   d) 20  
   e) 25
11. Let \( x = \log 2 \) and \( y = \log 3 \). Express \( \log \sqrt{72} \) in terms of \( x \) and \( y \).

   a) \( \sqrt{x^3 y^2} \)  
   b) \( \frac{x^3 y^2}{2} \)  
   c) \( \sqrt{3x + 2y} \)  
   d) \( \frac{3x + 2y}{2} \)  
   e) \( 3x + y \)

12. Two national park quarters are each flipped twice. What is the probability that each “park side” comes up exactly once?

   a) \( \frac{1}{16} \)  
   b) \( \frac{1}{8} \)  
   c) \( \frac{1}{4} \)  
   d) \( \frac{3}{8} \)  
   e) \( \frac{1}{2} \)

13. A sheriff’s star has seven points. Assume the star can be described by starting with seven equally-spaced points on a circle and then connecting, with a straight line segment, each point with the point adjacent to its neighbor. The angle measure in radians of each point is

   a) \( \frac{\pi}{7} \)  
   b) \( \frac{2\pi}{7} \)  
   c) \( \frac{3\pi}{7} \)  
   d) \( \frac{4\pi}{7} \)  
   e) \( \frac{7}{2\pi} \)

14. Let \( A = \{1, 2, 3\} \), \( B = \{4, 5, 6\} \), \( C = \{7, 8, 9\} \). Suppose \( E \), \( F \), and \( G \) disjoint sets such that each have exactly one element from each of \( A \), \( B \), \( C \). In how many ways is it possible to pick the sets \( E \), \( F \), and \( G \)?

   a) 196  
   b) 144  
   c) 216  
   d) 243  
   e) 256

15. The equations \( x = y \) , \( y = z \), and \( z = x \) represent planes in \( \mathbb{R}^3 \). How many regions in \( \mathbb{R}^3 \) are determined by these three planes?

   a) 4  
   b) 6  
   c) 8  
   d) 10  
   e) 12

16. Ninety-eight $1 bills are put in a hat. Also in the hat, are one $50 bill and one $100 bill. Lucky Lucy is allowed to reach in and grab one bill at random. The standard deviation \( \sigma \) of this experiment satisfies:

   a) \( 6 < \sigma < 8 \)  
   b) \( 8 < \sigma < 10 \)  
   c) \( 10 < \sigma < 12 \)  
   d) \( 12 < \sigma < 14 \)  
   e) \( 14 < \sigma < 16 \)
17. A radius of a circle is increased by one unit. What is ratio of the new area to the new circumference? Assume the original radius was \( r \).
   
   a) \( r+1:r \)  
   b) \( r+1:2 \)  
   c) \( \frac{(r+1)^2}{2} \)  
   d) \( \pi:2(r+1) \)  
   e) \( r+1:\pi \)

18. In the equation \( 23^2 + y^2 = z^2 \) it is known that \( y \) and \( z \) are positive integers. Then the value of \( y \) is
   
   a) 528  
   b) 166  
   c) 84  
   d) 220  
   e) 264

19. A standard die is placed on a standard chessboard. A turn consists of moving the die to an adjacent square (not diagonal). During the move the die is flipped along its corresponding edge. For example, if the die was placed so that 1 was showing on its top, and then the die was moved one square to the right, then the 1 would be on its right face and whatever had been on the left face will now be on top. If at the beginning the die is in the lower left corner, with 1 showing on top, what is the minimum number of moves to get to the upper right corner and have 1 showing on the top?
   
   a) 12  
   b) 14  
   c) 16  
   d) 18  
   e) 20

20. As usual, let \( \sqrt{-1} = i \). Consider the polynomial \( z^2 - (7+5i)z + 26 + 22i \). One root is \( 3+7i \). What is the other root?
   
   a) \( 4-2i \)  
   b) \( 6+2i \)  
   c) \( -3-i \)  
   d) \( 10+i \)  
   e) \( 5-4i \)

21. As in the previous question \( \sqrt{-1} = i \). Which of the following equals \( \frac{64+18i}{5+3i} \)?
   
   a) \( 11-5i \)  
   b) \( \frac{64}{5}+6i \)  
   c) \( \frac{266}{34} - 3i \)  
   d) \( \frac{374-282i}{34} \)  
   e) \( 11-3i \)
22. Suppose $2^x = 8^{x+1}$ and $3^y = 9^{x-2}$. Then $x - y$ equals:
   a) 1     b) 1.5     c) 2.2
   d) 2.7     e) 4

23. The equation $\sqrt{(x-3)^2 + y^2} + \sqrt{(x+3)^2 + y^2} = 10$ represents a region in the x-y plane. What is the area of the region?
   a) 20     b) $20\sqrt{2}$     c) $20\pi$
   d) $36\sqrt{3}$     e) $30\pi$

24. Suppose ABC is an isosceles triangle with right angle at B. Let D be the midpoint of BC and E be the midpoint of AB. Further let F be the point of intersection of medians AD and CE. What is the measure of $\angle AFE$?
   a) $\tan^{-1}(1/4)$     b) $\tan^{-1}(1/2)$     c) $\tan^{-1}(3/4)$
   d) $\tan^{-1}(2)$     e) $\tan^{-1}(2) - \tan^{-1}(1/4)$

25. Which of the following is equal to $\sin x + \sin 2x$ for all $x$ that are not multiples of $\pi$?
   a) $\frac{\sin 3x}{\sin 2x} \cdot \sin x$
   b) $\frac{\sin 3x}{\sin x} \cdot \sin \frac{x}{2}$
   c) $\frac{\sin \frac{3x}{2}}{\sin \frac{x}{2}} \cdot \sin x$
   d) $\frac{\sin \frac{3x}{2}}{\sin x} \cdot \sin 2x$
   e) None of these