For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the Examination. If you finish early, review your answers. If you finish early, you should double check your answers. When time is up and the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.
1. If the graphs of \( 5y + x + 3 = 0 \) and \( 3y + ax + 5 = 0 \) are to meet at right angles, the value of \( a \) is:
   a) \( \frac{3}{5} \)  
   b) \( -\frac{5}{3} \)  
   c) 15  
   d) 5  
   e) -15

2. One hundred sophomores took a test. Their average score was 70. The average of the top 30 students was 84. What was the average of the other 70 students?
   a) 60  
   b) 64  
   c) 65  
   d) 66  
   e) 68

3. A rectangle started with length \( x \) yards and width \( y \) yards. The length was tripled and then had an extra yard added. The width was quadrupled and then had two more yards added. Which of the following expressions represents the amount of area that was added to the original rectangle?
   a) \( 9x^2 + 16y^2 + 2 \)  
   b) \( 12xy + 6x + 4y + 2 \)  
   c) \( 11xy + 6x + 8y + 2 \)  
   d) \( 11xy + 6x + 4y + 2 \)  
   e) None of these

4. A pentagon has vertices at the points (-3,1), (2,1), (4,3), (3,5), (-4,4). What is its area?
   a) 24  
   b) 26  
   c) 27.5  
   d) 28  
   e) 30

5. Three people each draw a card from a standard deck of playing cards (without replacement). The probability that none of the cards match in rank (they can match in suit, but not in value – e.g. only one of them can be an ace or a 7, etc) can be written as \( \frac{p}{q} \). If this fraction is in lowest terms, what is \( p \)?
   a) 32  
   b) 212  
   c) 352  
   d) 672  
   e) 842
6. Jerry and his family drive 150 miles to the beach in three hours. On the way back, they take the scenic route and it takes them 5 hours. The difference between average going speed and their overall average speed, in mph, is

a) 7.5  
b) 8  
c) 10  
d) 12.5  
e) 15

7. To make 12 widgets, it takes 8 workers 16 days, when they work 6 hours each day. How many days would it take to make 30 widgets, if 12 workers were available and they each worked 8 hours per day?

a. 5  
b. 10  
c. 15

d. 20  
e. 25

8. A price of a dress was marked up by 30% and then, a month later it was marked up again by 30%. A year later it still had not sold. The shopkeeper marked it back down to the original price. This represented a discount of what percent of the latest price. Answer to the nearest whole percent

a) 40  
b) 41  
c) 52

d) 69  
e) 70

9. A right isosceles triangle has legs of length 1. A square is attached to the hypotenuse, outside the triangle. What is the distance from the right angle of the triangle to one of further vertices of the square?

a) 2  
b) \( \sqrt{5} \)  
c) \( \sqrt{6} \)

d) \( \sqrt{7} \)  
e) \( \sqrt{11}/2 \)

10. A radius of a circle is increased by one unit. What is ratio of the new area to the new circumference? Assume the original radius was \( r \).

a) \( \frac{r+1}{2} \)  
b) \( r+1 \)  
c) \( \frac{\pi r}{3} \)

d) \( 2(r+1) \)  
e) None of these
11. Consider \( A = \frac{x^3 y^{-2} z^4}{x^{-2} y^3 z^{-5}} \). If \( A \) is rewritten as a fraction with only positive exponents, then the sum of the exponents would be:

a) 16  

b) 17  

c) 18  

d) 19  

e) 20  

12. Simplify \( \frac{\log_2 7}{\log_2 \frac{1}{7}} \)

a) 0  

b) \( \log_2 49 \)  

c) \( \log_2 (\frac{7}{1}) \)  

d) 1  

e) -1  

13. The values of \( x \) that are more than 7 units away from 3 can be described using the absolute value function as:

a) \(|x-7| < 3\)  

b) \(|x-7| > 3\)  

c) \(|x+3| < 7\)  

d) \(|x-3| < 7\)  

e) \(|x-3| > 7\)  

14. Suppose two numbers are independently randomly chosen to be any real numbers between 0 and 1. What is the probability that their sum is less than \( \frac{1}{2} \)?

a) \( \frac{1}{2} \)  

b) \( \frac{1}{3} \)  

c) \( \frac{1}{4} \)  

d) \( \frac{1}{6} \)  

e) \( \frac{1}{8} \)  

15. Among 200 children, 70 are enrolled in math and 80 are enrolled in English. Let \( x \) be the number in neither subject. Then which of the following best describes the possibilities for \( x \)?

a) \( 50 \leq x \leq 150 \)  

b) \( 70 \leq x \leq 80 \)  

c) \( 50 \leq x \leq 100 \)  

d) \( 50 \leq x \leq 120 \)  

e) \( 50 \leq x \leq 130 \)
16. Given that \[-x + y + z = 10 \quad x + y - z = 8 \quad x - y + z = 6\], find \[x + y + z\]
   a) 20  b) 24  c) 32
   d) 40  e) 50

17. Consider the sum \[S = 1 + 2 + 3 + 4 + \ldots + n\]. What is the largest value of \(n\), for which \(S < 2011\) is true?
   a) 60  b) 61  c) 62
   d) 63  e) 64

18. One angle of a rhombus has measure 120 degrees. The side length is 4. What is the area of the rhombus?
   a) 16  b) \(4\sqrt{2}\)  c) \(8\sqrt{2}\)
   d) \(8\sqrt{3}\)  e) \(12\sqrt{3}\)

19. Josy put $100 in the bank. Every \(k\) years, the money grows by 100%. Relative to her original deposit, the money in the account after \(3k\) years represents an increase of what percent?
   a) 500  b) 700  c) 800
   d) 900  e) 1000

20. What is the geometric mean of the numbers 325 and 637
   a) 430  b) 455  c) \(15925/37\)
   d) 480  e) 481
21. The repeating decimal \(0.\overline{2636} = 0.2636363636...\) is a rational number and can be written as the quotient of two natural numbers \(\frac{p}{q}\). Assuming this fraction is in lowest terms, what is the value of \(p\)?

a) 17  
b) 19  
c) 29  
d) 37  
e) 59

22. Define two operations as follows: \(a \Delta b = \frac{2a + b}{3}\) and \(a \nabla b = \frac{a + 2b}{3}\). Simplify \((27 \Delta 81) \Delta (27 \nabla 81)\).

a) 51  
b) 57  
c) 45  
d) 87  
e) 33

23. Point P is outside a circle with center O. Points A, B, C, and D are on the circle such that P, A, and B are collinear with A in between P and B; and P, C, and D are collinear with C in between P and D. Given that \(\angle APC\) is 15 degrees and \(\angle BOD\) is 55 degrees, what is \(\angle AOC\)?

a) 20  
b) 25  
c) 27.5  
d) 40  
e) 42.5

24. The following inequality determines a region in the Cartesian plane: \(|x| + |y| + |x| - |y| \leq 10\). What is the area of this region?

a) 100  
b) 125  
c) 150  
d) 225  
e) 250

25. The first term of a geometric sequence is \(\sqrt{2011}\). The third term of the sequence is \(\sqrt[3]{2011}\). What is the fourth term?

a) \(\frac{12}{\sqrt{2011}}\)  
b) \(\frac{18}{\sqrt[3]{2011}}\)  
c) \(\sqrt[4]{\frac{1}{2011}}\)  
d) \(\frac{24}{\sqrt[3]{2011}}\)  
e) None of these