Theory of combined exciton-cyclotron resonance in a two-dimensional electron gas: The strong magnetic-field regime

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I developed a theory of combined exciton-cyclotron resonance (ExCR) in a low-density two-dimensional electron gas in high magnetic fields. In the presence of excess electrons an incident photon creates an exciton and simultaneously excites one electron to higher-lying Landau levels. I derived exact ExCR selection rules that followed from the existing dynamical symmetries, magnetic translations and rotations about the magnetic field axis. The nature of the final states in ExCR is elucidated. The relation between ExCR and shake-up processes is discussed. The double-peak ExCR structure for transitions to the first electron Landau level is predicted.

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Optical manifestations of many-body effects in low-dimensional electron-hole \((e-h)\) systems in magnetic fields have been the focus of many experimental and theoretical studies during the past decade. Recently, such objects as, e.g., artificial atoms in quantum dots and negatively-charged excitons \(X^-\) in quantum wells have been under intense scrutiny. The surprising apparent stability of the \(X^-\) (which in the dilute limit is a weakly-bound state \(^2\) of two electrons and one hole) in the presence of excess electrons in strong magnetic fields, and the relation of this stability to a many-body collective response have been actively discussed. Another interesting manifestation of many-body effects are shake-up processes in the photoluminescence of a two-dimensional electron gas (2DEG): After the recombination of the \(e-h\) pair, one electron is excited to one of the higher Landau levels (LL’s). A closely-related phenomenon, combined exciton-cyclotron resonance (ExCR) also has been identified in low-density 2DEG systems: Here, an incident photon creates an exciton and simultaneously excites one electron to higher LL’s. These phenomena and the relation between them remain only partially understood. A theory of ExCR has been developed for weak magnetic fields, when the magnetic length \(l_B = (\hbar e B/\epsilon_0)\) is much larger than the exciton Bohr radius \(\alpha_B = e^2/\epsilon_m^2 l_B \sim 1\). The energy positions of the ExCR spectra were obtained from an expansion in the unbound \(e^-+e^+\) states. The Coulomb interactions were taken into account phenomenologically as two-particle \(e^-h^+\) excitonic corrections to the transition matrix elements. In addition, the following assumptions were made: (i) a strictly-2D system, (ii) background electrons in the lowest spin-polarized \(n=0\) LL with spins oriented parallel to the field, (iii) low electron density \(n_e \approx 1\). In this work, I developed the theory of ExCR for the physically interesting regime of strong magnetic fields, \(l_B \sim \alpha_B\). Otherwise, I adopted essentially the same assumptions (i)–(iii), as in Ref. 5. Note that in the high-\(B\) limit, the characteristic length of the problem is \(l_B\) rather than \(\alpha_B\), and condition (iii) can be formulated in terms of the filling factor, as \(\nu_e = 2\pi \theta_B n_e \ll 1\). In the limit of low-electron density, ExCR can be considered to be a three-particle resonance involving a \(\text{charged system}\) of two electrons and one hole, \(2e^-h^+\), in the final state. Importantly, there is a coupling of the center-of-mass and internal motions for charged \(e^-h^+\) complexes in magnetic fields. In order to describe the high-field ExCR, I obtained the complete spectra of the \(2e^-h^+\) eigenstates in higher LL’s with a consistent treatment of the Coulomb correlations. I exploited a recently developed scheme for charged \(e^-h^+\) complexes in magnetic fields in which one degree of freedom is separated while all existing dynamical symmetries, rotations about the axis and magnetic translations, are preserved. This allows one to establish exact ExCR selection rules that are applicable in arbitrary magnetic fields, to derive the ExCR oscillator strengths, and to establish the heretofore missing relation between ExCR and shake-up processes in the dilute limit.

The Hamiltonian describing the \(2D\) \(2e^-h^+\) state in a perpendicular magnetic field \(B=(0,0,B)\) is given by

\[
H = \sum_{i=1,2} \frac{\hat{p}_{ri}^2}{2m_e} + \sum_{i=1,2} \frac{\hat{p}_{r_i}^2}{2m_h} - \sum_{i=1,2} \frac{e^2}{|r_i-r_j|} + \frac{e^2}{|r_1-r_2|},
\]

where \(\hat{p}_{r_i} = -i\hbar \nabla_r - e A(r)/c\) are kinematic momentum operators and the symmetric gauge \(A = \frac{1}{2} B \times r\) is used; a weak exchange \(e^-h^+\) interaction, small central-cell corrections to the Coulomb potential, and the crystal anisotropy are not relevant for the present study and are neglected. The exact eigenstates of (1) form families of degenerate states; each family is labeled by the index \(\nu\) that plays a role of the principal quantum number and can be discrete (bound states) or continuous (bound states forming a continuum). There is a macroscopic number of degenerate states in each family labeled by the discrete oscillator quantum number \(k = 0,1,\ldots\). This quantum number is associated with magnetic translations and physically describes the center-of-rotation of the charged complex in \(B\). Each family starts with its parent state \((PS)\) \(|\Psi_{k=0} M_{S_z} S_h\rangle\) that has \(k = 0\) and the largest (for the total charge \(Q<0\)) possible in the family value of the total angular momentum projection \(M_z\). Degenerate daughter states \(|\Psi_{k=1} M_{S_z} S_h\rangle\) can be constructed iteratively out of the PS. \(S_{z}\) denotes the total spin of two electrons, either \(S_{z}=0\) (singlet states) or \(S_{z}=1\) (triplet states); \(S_h\) is the spin state of the hole. Noninteracting electrons (or holes) in \(B\) can also be classified according to

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this scheme. The corresponding $e$- and $h$- single-particle wave functions $\psi_{nm}(r) = \psi_{nm}(r)$ have the factored form and are well-known in the theory of the fractional quantum Hall effect: \[^{10}\] $n$ is the LL quantum number, which determines the energy $\hbar \omega_{c}(n + \frac{1}{2})$ with $\omega_{c} = eB/m_{e}c$, and $m = 0, 1, \ldots$ is the intra-LL oscillator quantum number, the single-particle version of $k$. The angular momentum projections are $m_{e} = n - m$. In the zero LL

$$\psi_{nm}(r) = \frac{1}{\sqrt{2\pi l_{B}^{2}m!}} \left( \frac{z^{*}}{\sqrt{2l_{B}}} \right)^{m} \exp\left( -\frac{r^{2}}{4l_{B}^{2}} \right),$$

where $z^{*} = x - iy$.

Let us derive the optical selection rules for ExCR. Interband transitions with $e$-$h$ pair creation can be described by the luminescence operator $\hat{\mathcal{L}}_{PL} = p_{cv} \int d\mathbf{r} \hat{\psi}_{e}^{*}(\mathbf{r}) \hat{\psi}_{h}^{*}(\mathbf{r})$, where $p_{cv}$ is the interband momentum matrix element. Consider the creation of the $e$-$h$ pair in the presence of the low-density 2DEG in the $n$-th LL (see Fig. 1). Taking into account only the three-particle correlations in the final state, the dipole transition matrix element is

$$D(v) = \langle \Psi_{kM_{z} = kS_{e}S_{h}v} | \hat{\mathcal{L}}_{PL} | \psi_{nm} \rangle \rangle.$$  

The oscillator quantum number is conserved in dipole transitions: $m = k$. Physically this means that the center-of-rotation of charged complexes in the initial and final states must coincide. Due to the change of the Bloch wave functions, the usual selection rule $\Delta M_{z} = 0$ holds for the envelope functions, thus, $m_{e} = n - m = M_{z} - k$. The combination of the two selection rules leads to $D(v) \sim \delta_{h,M_{z}}$, where $M_{z}$ is the angular momentum projection of the PS in the $n$th family. Therefore, in the ExCR processes involving electrons from the $n$th LL, the achievable final $e$-$h$ states must have $M_{z} = n$ and may belong to different final LL’s. If the 2DEG is spin-up $\uparrow$ polarized, the photon of $\sigma^{+}$ ($\sigma^{-}$) circular polarization produces the singlet (triplet) final states (see Fig. 1).

It is illuminating to compare the ExCR processes and photoluminescence (PL) of negatively-charged excitons, $X^{-} \rightarrow e_{n}^{-} + photon$, in which the electron is left in the $n$th LL in the final state. Such processes are described by the transition matrix element $D_{u}(v) = \langle \psi_{nm} | \hat{\mathcal{L}}_{PL} | \Psi_{kM_{z} = kS_{e}S_{h}v} \rangle$ and can be considered as the inverse of ExCR. The exact selection rule $D_{u}(v) \sim \delta_{M_{z}, n}$ shows that the $X^{-}$ PL transition is only possible when the electron is left in a single and specific LL with the number $n = M_{z}$. Therefore, contrary to ExCR, no various final LL’s are achievable in the PL from any given $X^{-}$ state. In this sense, the shake-up processes, \(^{4}\) having as the final states various LL’s $n = 1, 2, \ldots$ must be prohibited in the PL of the isolated $X^{-}$ in a translationally invariant system in $B$.

Below, I studied ExCR from the zero spin-polarized $n = 0 \uparrow$ LL to the first electron LL in high magnetic fields

$$\hbar \omega_{ce}, \hbar \omega_{ch} \gg E_{0} = \sqrt{\frac{\pi}{2}} \frac{\epsilon^{2}}{e^{2}l_{B}}.$$  

The $2e$-$h$ states can be obtained in this regime as an expansion in LL’s. The complete orthonormal basis compatible with both axial and translational symmetries has been constructed in Ref. 8 and will be denoted here as

$$|n_{R}n_{e}n_{h}; kM_{z} \rangle.$$  

The conserved oscillator quantum number is fixed and equals $k$ in (5) and $M_{z} = n_{R} + n_{e} - n_{h} - k - m + 1$. $n_{R}$ is the hole LL number, $n_{e}$ and $n_{h}$ are the LL numbers of the electron relative and center-of-mass motions, respectively (see Ref. 8 for details). The permutational symmetry requires that $n_{e} - m$ should be even (odd) for $S_{e} = 0$ ($S_{e} = 1$). Fixing $k$ in (5) amounts to summing the infinite number of $e$- and $h$- states in zero LL’s. As an example, the state $|\vec{0}\rangle = |000; 000\rangle$ — the new vacuum — has the form

\[
|\vec{0}\rangle = \sum_{n_{R}n_{e}n_{h}} |n_{R}n_{e}n_{h}; kM_{z} \rangle.
\]
and is a coherent \( e-h \) state [cf. Eq. (2)]. In what follows I consider only the PS’s \( |\Psi_{M_x(t)\psi}\rangle \), where the quantum numbers \( k = 0 \) and \( s_x \) are omitted for brevity and \( s(t) \) denotes the singlet \( s_x = 0 \) (triplet \( s_x = 1 \)) electron spin state.

Neglecting mixing between LL’s (the high-field limit), the triplet \( e-h \) states in the first electron and zero hole LL, \( |\Psi_{M_x(t)}^{(10)}\rangle \), can be obtained as the expansion

\[
|\Psi_{M_x(t)}^{(10)}\rangle = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \alpha_{M_x(t)}(2m,l)|000;02ml\rangle \\
+ \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \beta_{M_x(t)}(2m+l,1)|100;02m+1l\rangle,
\]

where expansion coefficients \( \alpha_{M_x(t)}(2m,l) \sim \delta_{M_x,l+1-2m} \) and \( \beta_{M_x(t)}(2m+l,1) \sim \delta_{M_x,l-2m} \). The Coulomb matrix elements of the Hamiltonian (1) are calculated analytically and the eigenspectra are obtained by numerical diagonalization of finite matrices of order \( 2 - 5 \times 10^2 \). Such finite-size calculations provide very high accuracy for bound \( X^- \) states and are also capable of reproducing the structure of the three-particle continuum. The singlet states \( |\Psi_{M_x(t)}^{(10)}\rangle \) are obtained by using a similar procedure.

The calculated triplet \( e-h \) eigenspectra are presented in Fig. 2. Filled dots above free LL’s show positions of the excited bound three-particle states, denoted as \( (2e-h) \). These states originate from the excited states of two electrons that are bound in 2D because of the confining effect of the magnetic field. Bound \( (2e-h) \) states appear in the spectrum for relatively large values of \( M_x \), when the hole can be at a sufficiently large distance from the two electrons. There is also exactly one low-lying triplet bound state — the negatively-charged magnetoexciton (MX) \( X_{10}^- \) that has \( M_z = 1 \) and binding energy \( 0.086E_0 \). The shaded area of width \( E_0 \) in Fig. 2 corresponds to the three-particle continuum formed by the neutral MX \( X_{10} \) \( (e \text{ and } h \text{ in their zero LL’s}) \) with the second electron in a scattering state in the first LL. The hatched area corresponds to the second overlapping band formed by the states of the neutral MX \( X_{10} \) \( (e \text{ in the first and } h \text{ in the zero LL}) \) with the second electron in a scattering state in the zero LL. The lower continuum edge lies at the \( X_{10} \) ground-state energy \( -0.574E_0 \), which, for the isolated MX, is achieved at a finite center-of-mass momentum \( |\mathbf{k}|/\hbar \approx 1.19 \). As a result, the density of \( X_{10} \) states has at this energy an inverse square-root van Hove singularity in 2D. The continuum of the singlet \( 2e-h \) states has qualitatively the same structure; there are also bound singlet \( (2e-h) \) states above free LL’s (not shown). There are, however, no low-lying bound singlet \( X_{10} \) states.

Due to the selection rule \( D(v) \sim \delta_{h,M_x} \), the \( 2e-h \) states with \( M_x = 0 \) are active in the ExC transitions from the \( n = 0 \) LL. As a result, the ExC transition \( e_0^+ + \text{photon} \rightarrow X_{10}^+ \) to the bound negatively-charged triplet MX is prohibited. ExC transitions to the bound excited triplet and singlet \( (2e-h) \) states are also prohibited: all these states have large \( M_x \). Therefore, the ExC transitions from zero to the first electron LL can only go to the continuum.

All ExC transitions are only due to LL mixing. In order to calculate the ExC dipole transition matrix elements, I went beyond the high-field limit and admix to (7) the triplet \( 2e-h \) states in zero LL’s \( \sum_{m=0}^{\infty} \gamma_{M_x,v}(2m+1,l)|000;02m+1l\rangle \) and \( \gamma_{M_x,v}(2m+1,l) \sim \delta_{M_x,l-2m-1} \) and \( \gamma_{M_x,v}(2m+1,l) \sim \delta_{M_x,l-2m} \). The coordinate representation has the form

\[
\Phi_{m}(r_1,r_2,r_3) = \langle r_1,r_2,r_3 | 000;02m \rangle = \frac{1}{\sqrt{m!}} \left( \frac{\hbar}{2l_B} \right)^{m} \frac{z_h}{2l_B} \Phi_{0}(r_1,r_2,r_3).
\]

A similar procedure is used for the singlet \( |\Psi_{M_x(t)}^{(10)}\rangle \) states.

The dipole transition matrix element is given by

\[
D_i(v) = \langle \Psi_{M_x(t)}^{(0)} | \hat{E}_p | \phi_{0}^{(e)} \rangle = \rho_C \sum_{m=0}^{\infty} \gamma_{M_x,v}(2m+1,2m+1) D_{2m+1},
\]

\[
D_p = \int dr_1 \int dr_2 \Phi^{(e)}_{pp}(r_1,r_2) \phi_{0}^{(e)}(r_1).
\]

Using Eqs. (2), (6), and (8) and performing in (10) the transformation \( z_2^+ \rightarrow z_2^- \) in the complex \( (z_2,\bar{z}_2) \) plane, it can be shown that the overlap integral \( \rho_p = (1)^{p+2} \). The intensity of the ExC transitions involving the \( 2e-h \) states with eigenenergies \( E_v \) is

\[
I_{ExC}^{(e)}(\omega) \sim \nu, \sum_{p} |D_i(v)|^2 \delta(h \omega - E_v).
\]
Surprisingly, the present theory predicts in high fields an additional strong feature—the second, higher-lying, peak in the ExCR. If this peak were simply due to the transitions to higher LL’s, may have multiple-peaks and complicated lineshapes because of more involved structures of the continua.

The intensity of the ExCR lower peak is larger in the $\sigma^-$ polarization (Fig. 3), which is consistent with experiment. The present theory predicts that the intensity of the second ExCR peak will show the opposite dependence: It should be larger in the $\sigma^+$ polarization. This polarization dependence is due to different $e^-e^-$ correlations in the final singlet ($\sigma^+$ polarization) and triplet ($\sigma^-$ polarization) $2e-h$ states: The oscillator strength is transferred to the higher lying peak when the final singlet $2e-h$ states—characterized by a larger $e^-e^-$ repulsion—are involved. In agreement with Ref. 5, the present theory shows that (1) the ExCR peaks have intrinsic finite linewidths, in high fields $\sim 0.15E_0(\sim 2.6$ meV at $B \approx 10$ T corresponding to Fig. 3), and have asymmetric lineshapes with high-energy tails; (2) the ExCR transitions are because of LL mixing and, therefore, ExCR is suppressed in strong fields as $r_1D^2 \sim n_e^2(l_B/a_B)^2B^{-2}$.

In conclusion, a magneto-optical phenomenon, ExCR transitions in low-density 2DEG systems, has been considered theoretically for high magnetic fields. The developed formalism allows a consistent treatment of the final state $e^-h$ and $e^-e^-$ Coulomb interactions. The features of the high-field ExCR, in particular, the double-peak structure of the transitions to the first electron Landau level have been predicted.

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6 On leave from General Physics Institute, RAS, Moscow 117942, Russia.
9 This is quantitatively correct at low magnetic fields, when the bound complex rotates in $B$ as a whole.
12 This conclusion is reached for a strictly-2D system in the limit of high magnetic fields. It is applicable to quasi-2D systems at finite fields—as long as no bound $X^-$ states with $M_z=0$ appear in the first electron LL.