MATH 305 Spring 2009 Quiz 2 Name: Answers

1. Let $x = 2^{30} + 2^8 + 2^4$. Find the machine numbers on the MARC-32 that are just to the right (x^+) and just to the left (x^-) of x.

$$X = 2^{30} \times \left[1 + 2^{-22} + 2^{-26} \right]$$

$$= (1.000 \cdots 00\underset{\substack{\uparrow \\ 22}}{1}\underset{\substack{\uparrow \\ 23}}{0} \vdots 001)_2 \times 2^{30}$$

$$\therefore \quad X_- = (1.000 \cdots 00\underset{\substack{\uparrow \\ 22}}{1}0)_2 \times 2^{30} = 2^{30} + 2^8$$

$$X_+ = (1.000 \cdots 00\underset{\substack{\uparrow \\ 22}}{1}\underset{\substack{\uparrow \\ 23}}{1})_2 \times 2^{30} = 2^{30} + 2^8 + 2^7$$

$$= 2^{30} + 2^8 \qquad\qquad = 2^{30} + 2^8 + 2^7$$

$$x^- = \boxed{(1.000 \cdots 00\overset{\substack{22 \\ \downarrow}}{1}0)_2 \times 2^{30}} \qquad x^+ = \boxed{(1.000 \cdots 00\overset{\substack{22 \\ \downarrow}}{1}\overset{\substack{23 \\ \downarrow}}{1})_2 \times 2^{30}}$$

2. If at most 3 significant binary bits are to be lost in the computation $\sqrt{x^2+1} - 1$, what restriction must be placed on x?

$$\underbrace{2^{-3} \leqslant 1 - \frac{1}{\sqrt{x^2+1}}} < 2^{-p} \qquad (\text{for some } p < 3)$$

$$\Downarrow$$

$$\frac{1}{\sqrt{x^2+1}} \leqslant 1 - 2^{-3} = \frac{7}{8}$$

$$\therefore \quad \sqrt{x^2+1} \geqslant \frac{8}{7} \quad \Rightarrow \quad x^2 + 1 \geqslant \frac{64}{49}$$

$$\therefore \quad x^2 \geqslant \frac{15}{49}$$

$$\therefore \quad |x| \geqslant \frac{\sqrt{15}}{7} \approx 0.55328$$

Answer: $\boxed{|x| \geqslant \frac{\sqrt{15}}{7} \approx 0.55328}$