MATH 305 Spring 2009 Quiz 1 Name: Answers

1. For small values of x the approximation $\sin x \approx x - \frac{x^3}{6}$ can be used. For what range of values of x will this approximation give correct results rounded to 10 decimal places?

$$\sin x = x - \frac{x^3}{6} + \frac{\cos \xi}{120} \cdot x^5$$

$$\left| \sin x - x + \frac{x^3}{6} \right| \le \frac{1}{120} |x|^5$$

correct to 10 decimal places : $\frac{|x|^5}{120} \le \frac{1}{2} 10^{-10} \iff |x|^5 \le 60 \cdot 10^{-10}$

$$\therefore \quad |x| \le 60^{\frac{1}{5}} \cdot 10^{-2} \approx 0.0226793$$

Answer: $\boxed{\begin{array}{c} \left(-\sqrt[5]{60} \cdot 10^{-2} \le x \le \sqrt[5]{60} \cdot 10^{-2} \right) \\ -0.02268 \le x \le 0.02268 \end{array}}$

2. Find the Taylor polynomial of degree 12 for $f(x) = e^{x^4}$ expanded about 0.

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

$$e^{x^4} = \underbrace{1 + x^4 + \frac{x^8}{2} + \frac{x^{12}}{6}}_{\text{answer}} + \frac{x^{16}}{24} + \cdots$$

Answer: $\boxed{1 + x^4 + \frac{x^8}{2} + \frac{x^{12}}{6}}$