MATH 305 **LAB 4** **Spring 2009**

Name : _____ Partners : _____

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INSTRUCTIONS :

1. Write the names of you and your partners.
2. Write all intermediate steps, and circle the answers.
3. Answers given without sufficient work to support them may NOT receive credit.
4. Every member of a group should turn in the lab report stapled with this paper on top.

Download Newton.m from the course website to your desktop. Run Matlab program. In the command window of Matlab. Type `f=inline('x.^3-3*x.^2+4/3');` after `>>` sign and enter, and type `df=inline('3*x.^2-6*x');` after `>>` sign and enter. Then, type `[x,y]=Newton(f,df,0.5,0.00001,20);` after `>>` sign and enter. The output you get is the result of Newton's method to the equation $x^3-3x^2+\frac{4}{3}=0$ with initial guess 0.5, tolerance 0.00001, and maximum number of iteration 20.

1. Let α_n denote the n^{th} positive root of the equation $\tan\left(x-\frac{\pi}{2}\right)=-x$.
   (a) Use Newton.m to find α_1, α_2, α_3, α_4. You need to change inline function f and df and need to choose an initial guess carefully for each case.
   (b) Can you guess $\lim_{n\to\infty}\frac{\alpha_n}{n}$? Explain your answer.

2. (a) Show that the equation $x^3-3x^2+x+3=0$ has a root in $(-1,2)$
   (b) Apply Newton's method with the initial guess $x_1=1$. Discuss about what you observe.
   (c) Find the root of the equation accurate to 8 decimal places.

3. Consider a sequence $\{x_n\}$ generated by Newton's method applied to the equation $\tan^{-1}(x)=0$.
   (a) Apply Newton's method to the equation with $x_0=0.5, 1, 1.3, 1.4, 2, 3$. What do you observe?
   (b) Guess the largest suitable positive initial guess accurate to three decimal places.
   (c) Show that if $x_0\neq 0$, then $x_{n+1}x_n<0$ for all $n\geq 0$.
   (d) Find a condition on x_n for which $|x_{n+1}|<|x_n|$.
   (e) Use (d) to find a condition on suitable initial guesses. Compare it with the result of (b).