MATH 305 **LAB 1** **Spring 2009**

Name: ______________________ Partners: ______________________

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INSTRUCTIONS:

1. Write the names of you and your partners.
2. Write all intermediate steps, and circle the answers.
3. Answers given without sufficient work to support them may NOT receive credit.
4. Every member of a group should turn in the lab report stapled with this paper on top.

1. Do #10 on page 13.

2. Do #30 on page 14.

3. Do #33 on page 14.

4. Let $a \in \mathbb{R}$ and $f, g : \mathbb{R} \to \mathbb{R}$. Suppose that f and g are continuously differentiable up to order $n + 1$. Use the Lagrange Remainder Theorem to prove that if f and g have contact of order n at a, then

$$\lim_{x \to a} \frac{f(x) - g(x)}{(x - a)^n} = 0$$

5. Use the Mean Value Theorem for Integrals (the lecture note version) to show that the Cauchy Remainder Theorem implies the Lagrange Remainder Theorem, if $f \in C^{n+1}(I)$.

6. Consider $f(x) = \ln(1+x)$ for $x \in (-1, 1]$.

(a) Find the nth Taylor Polynomial of f, $p_n(x)$ at 0.

(b) Use the Lagrange Remainder Theorem to show that $\lim_{n\to\infty} p_n(x) = f(x)$ for all $x \in [0, 1]$. Can you prove the convergence for $-\frac{1}{2} \le x < 0$? How about the case of $-1 < x < -\frac{1}{2}$?

(c) Use the Cauchy Integral Remainder Theorem to show that for all $x \in (-1, 1]$ and all $n \in \mathbb{N}$,

$$R_{n+1}(x) = f(x) - p_n(x) = \int_0^x \frac{(t-x)^n}{(1+t)^{n+1}}\, dt$$

(d) Show that for all $-1 < x \le t \le 0$,

$$0 \le \frac{t-x}{1+t} \le \frac{-xt-x}{1+t} = -x$$

(e) Use (c) and (d) to prove that for all $-1 < x \le t \le 0$ and all $n \in \mathbb{N}$,

$$|R_{n+1}(x)| \le |x|^n \int_x^0 \frac{1}{1+t}\, dt = -\,|x|^n \ln(1+x)$$

(f) Use (e) to show that $\lim_{n\to\infty} p_n(x) = f(x)$ for all $x \in (-1, 0)$.