HW 6.2

3. $f[x_0, x_1, \cdots, x_n] = \dfrac{f^{(n)}(\xi)}{n!}$

where $\xi \in I(x_0, x_1, \cdots, x_n) \subset [a,b]$

If $x_i \to x_0$, $\forall i = 1, 2, \cdots, n$,

then $\xi \to x_0$ also

Since $f^{(n)}$ is conti on $[a,b]$

$(\because f \in C^n[a,b])$ $f^{(n)}(\xi) \to f^{(n)}(x_0)$

$\therefore \lim\limits_{\substack{x_i \to x_0 \\ i=1,\cdots,n}} f[x_0, x_1, \cdots, x_n] = \dfrac{f^{(n)}(x_0)}{n!}$

4. $f \in \mathbb{P}_k$, where $k < n$

$\Rightarrow f^{(n)}(x) \equiv 0$

$\therefore f[x_0, x_1, \cdots, x_n] = \dfrac{f^{(n)}(\xi)}{n!} = 0$

5. If $p \in \mathbb{P}_n$, then the interpolation of p at $x_0, x_1, \cdots, x_n$ is itself.
(by the uniqueness of the interpolation)

In Newton's form

$$P(x) = \sum_{i=0}^{n} P[x_0, x_1, \cdots, x_i] \prod_{j=0}^{i-1} (x - x_j)$$

8. The Lagrange form of the interpolation of f at $x_0, x_1, \cdots, x_n$

$$= \sum_{i=0}^{n} f(x_i)\, l_i(x)$$

A Newton form of the interpolation of f at $x_0, x_1, \cdots, x_n$

$$= \sum_{i=0}^{n} f[x_0, x_1, \cdots, x_i] \prod_{j=0}^{i-1} (x - x_j)$$

By the uniquness of the interpolation

$$\underbrace{\sum_{i=0}^{n} f(x_i)\, l_i(x) = \sum_{i=0}^{n} f[x_0, x_1, \cdots x_i] \prod_{j=0}^{i-1} (x - x_j)}_{(1)}$$

9. Compare the coefficient of x^n in (1).

From RHS : $f[x_0, x_1, \cdots, x_n]$

From LHS : $\sum_{i=0}^{n} f(x_i) \prod_{\substack{j=0 \\ j \neq i}}^{n} (x_i - x_j)^{-1}$

(Also, see #10 on P324)

$\therefore f[x_0, x_1, \cdots, x_n] = \sum_{i=0}^{n} f(x_i) \prod_{\substack{j=0 \\ j \neq i}}^{n} (x_i - x_j)^{-1}$

22. $P_3(x) = 51 - 48x + 23x(x-1) - \frac{16}{7} x(x-1)(x-2)$

23. $P(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1) + c\, x(x-1)(x+1)(x-2)$

Then,

$10 = P(3) = 2 - 4 + 12 - 2 \cdot 3 \cdot 4 \cdot 2 + c \cdot 3 \cdot 2 \cdot 4 \cdot 1$

$\therefore \cancel{10} = \cancel{10} - 48 + 24c$

$\therefore c = 2$

$\therefore P(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1) + 2x(x-1)(x+1)(x-2)$

24. $P(x) = 63 + 26(x-4) + 6(x-4)(x-2) + x(x-4)(x-2)$