

$$\begin{aligned}
 2. \quad x_{n+1} &= x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \\
 &= x_n - f(x_n) \cdot \frac{1}{\frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}} \quad \text{MVT} \\
 \therefore x_{n+1} &= x_n - \frac{f(x_n)}{f'(\xi_n)} \quad \text{for some } \xi_n \in I(x_{n-1}, x_n) \\
 \downarrow \quad \downarrow \quad \downarrow \\
 \xi &= \xi - \frac{f(\xi)}{f'(\xi)}, \quad \left(\begin{array}{l} \because f'(\xi) \neq 0 \\ \& \xi_n \rightarrow \xi, \text{ as } n \rightarrow \infty \end{array} \right) \\
 \text{as } n \rightarrow \infty.
 \end{aligned}$$

$$\therefore f(\xi) = 0$$

$$\begin{aligned}
 4. \quad x_2 &= 1 - f(1) \cdot \frac{1-0}{f(1)-f(0)} = 1 - (-1) \cdot \frac{1}{-1-(-2)} \\
 &= 1 + 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 5. \quad x_2 &= 2 - (1.5) \cdot \frac{2-1}{1.5-2} = 2 - 1.5(-2) \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 6. \quad (b) \quad \lim_{n \rightarrow \infty} \frac{x_n^c}{y_n^c} &= \lim_{n \rightarrow \infty} \left(\frac{x_n}{y_n} \right)^c \\
 &= \left(\lim_{n \rightarrow \infty} \frac{x_n}{y_n} \right)^c \quad \left(\begin{array}{l} \because f(x) = x^c \quad (c \neq 0) \\ \text{is continuous } \forall x \neq 0 \end{array} \right)
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad \lim_{x \rightarrow \infty} \frac{x_n}{v_n} &= \lim_{x \rightarrow \infty} \frac{x_n}{y_n} \cdot \frac{y_n}{u_n} \cdot \frac{u_n}{v_n} \\
 &= \left(\lim_{n \rightarrow \infty} \frac{x_n}{y_n} \right) \left(\lim_{n \rightarrow \infty} \frac{y_n}{u_n} \right) \cdot \left(\lim_{n \rightarrow \infty} \frac{u_n}{v_n} \right) \\
 &= 1 \cdot 1 \cdot 1 = 1 \\
 \therefore x_n &\sim v_n
 \end{aligned}$$

$$\begin{aligned}
 7. \quad x_{n+1} &= x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \\
 &= \frac{x_n (f(x_{n-1}) - f(x_{n+1}))}{f(x_n) - f(x_{n-1})} - \frac{f(x_n) (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \\
 &= \frac{f(x_n) x_{n-1} - f(x_{n+1}) x_n}{f(x_n) - f(x_{n-1})} \quad (*)
 \end{aligned}$$

(3) calculates x_{n+1} by adding "small" correction term to x_n , while (*) calculates x_{n+1} from the "scratch" every time.