HW 2.3 & 3.1

< Sec 2.3 >

4. $f(x) = x^\alpha \quad f'(x) = \alpha x^{\alpha-1}$

$(\text{cond } f)(x) = \left|\frac{x f'(x)}{f(x)}\right| = \left|\frac{\alpha x^\alpha}{x^\alpha}\right| = |\alpha|$

5. (a) $(\text{cond } f)(x) = \left|\frac{x\,\alpha\,(x-1)^{\alpha-1}}{(x-1)^\alpha}\right|$

$= \left|\frac{\alpha x}{x-1}\right|$

(b) $(\text{cond } f)(x) = \left|\frac{x \cdot \frac{1}{x}}{\ln x}\right| = \left|\frac{1}{\ln x}\right|$

(c) $(\text{cond } f)(x) = \left|\frac{x\cos x}{\sin x}\right| = |x \cot x|$

(d) $(\text{cond } f)(x) = \left|\frac{x e^x}{e^x}\right| = |x|$

< Sec 3.1 >

1. $x \approx 1.9$

3. Let r be the root

$\therefore\ 128 \le r \le 129$

If we have $|c_n - r| < 10^{-6}$

then $\left|\frac{c_n - r}{r}\right| < \frac{10^{-6}}{128} \approx 7.8 \times 10^{-9}$

i.e., the relative error is 7.8×10^{-9}.

However, the unit roundoff error ε in Marc-32 is $2^{-24} \approx 5.96 \times 10^{-8}$ which is quite bigger than 7.8×10^{-9}

$\therefore$ It is not possible to compute the root with absolute accuracy less than 10^{-6}.

4. To get $|r - c_n| \le \varepsilon$, we set

$2^{-(n+1)}(b_0 - a_0) \le \varepsilon$

$\Leftrightarrow\ 2^{-(n+1)} \le \frac{\varepsilon}{b_0 - a_0}$

$\Leftrightarrow\ 2^{n+1} \ge \frac{b_0 - a_0}{\varepsilon}$

$\therefore\ \ln(2^{n+1}) \ge \ln\left(\frac{b_0 - a_0}{\varepsilon}\right)$

$\Leftrightarrow\ (n+1)\ln 2 \ge \ln(b_0 - a_0) - \ln\varepsilon$

$\therefore\ n \ge \frac{\ln(b_0 - a_0) - \ln\varepsilon}{\ln 2} - 1$

7. $2 \le r \le 3$

(i) absolute accuracy $< 10^{-6}$

$|r - c_n| \le 2^{-(n+1)}(3-2) < 10^{-6}$

$\therefore\ 2^{-(n+1)} < 10^{-6}$

$-(n+1)\ln 2 < -6\ln 10$

$n+1 > \frac{6\ln 10}{\ln 2}$

$\therefore\ n > \frac{6\ln 10}{\ln 2} - 1 \approx 18.93$

$\therefore$ 19 steps

(ii) relative accuracy $< 10^{-6}$

$\frac{|r - c_n|}{|r|} < \frac{2^{-(n+1)}(3-2)}{2} < 10^{-6}$

$2^{-n-2} < 10^{-6}$

$(-n-2)\ln 2 < -6\ln 10$

$\therefore\ n+2 > \frac{6\ln 10}{\ln 2}$

$\therefore\ n > \frac{6\ln 10}{\ln 2} - 2 \approx 17.93$

$\therefore$ 18 steps