MATH 205 Spring 2009 Quiz 4 Name: Answer

1. Find the general solution to the following DE

$$y^{(4)} + 2y^{(3)} + 10y'' - 6y' + 65y = 0$$

(Hint: $e^x \sin 2x$ is a solution.)

ch. eq : $\underbrace{r^4 + 2r^3 + 10r^2 - 6r + 65}_{= P(r)} = 0$

$e^x \sin 2x$ is a solution

$\Longleftrightarrow$ $\left[\begin{array}{l} (r-1)^2 + 2^2 = r^2 - 2r + 5 \\ \text{is a factor of } P(r) \end{array} \right]$

$$\begin{array}{r} r^2 + 4r + 13 \\ r^2 - 2r + 5 \overline{\smash{\big)}\, r^4 + 2r^3 + 10r^2 - 6r + 65} \\ \underline{r^4 - 2r^3 + 5r^2} \\ 4r^3 + 5r^2 - 6r \\ \underline{4r^3 - 8r^2 + 20r} \\ 13r^2 - 26r + 65 \\ \underline{13r^2 - 26r + 65} \\ 0 \end{array}$$

$\therefore$ $P(r) = (r^2 - 2r + 5)(r^2 + 4r + 13)$

$\therefore$ $P(r) = 0$

$\Longleftrightarrow$ $r^2 - 2r + 5 = 0$ or $r^2 + 4r + 13 = 0$

$\therefore$ $r = 1 \pm 2i$ or $-2 \pm 3i$

$y(x) = \boxed{c_1 e^x \cos 2x + c_2 e^x \sin 2x + c_3 e^{-2x} \cos 3x + c_4 e^{-2x} \sin 3x}$