

1. Solve the DE

$$\frac{dy}{dx} = \frac{x^2 - y^2}{3xy}$$

$$\frac{dy}{dx} = \frac{1}{3} \frac{1}{\frac{y}{x}} - \frac{1}{3} \frac{y}{x} \quad : \text{Homogeneous eq}$$

$$v \equiv \frac{y}{x} \rightarrow y = xv \rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{1}{3} \frac{1}{v} - \frac{1}{3} v \rightarrow x \frac{dv}{dx} = \frac{1}{3} \frac{1}{v} - \frac{4v}{3} = \frac{1-4v^2}{3v}$$

$$\therefore \int \frac{3v dv}{1-4v^2} = \int \frac{dx}{x} \rightarrow -\frac{3}{8} \int \frac{-8v dv}{1-4v^2} = \int \frac{dx}{x}$$

$$\therefore -\frac{3}{8} \ln |1-4v^2| = \ln|x| + C$$

$$\therefore 3 \ln |1-4v^2| = -8 \ln|x| + C$$

$$3 \ln \left| 1 - 4 \frac{y^2}{x^2} \right| + 8 \ln|x| = C$$

$$\ln \left| \left( 1 - \frac{4y^2}{x^2} \right)^3 x^8 \right| = \tilde{C}$$

$$\therefore \left( 1 - \frac{4y^2}{x^2} \right)^3 x^8 = C$$

$$\therefore x^2 (x^2 - 4y^2)^3 = C$$

Answer:

$$x^2 (x^2 - 4y^2)^3 = C$$