MATH 205 Spring 2009 Quiz 1 Name: Answer

1. Solve the IVP

$$(x^2 + x^2y^2)\,dx + e^{x^3}y\,dy = 0, \quad y(0) = 0.$$

$$x^2(1+y^2)\,dx + e^{x^3}y\,dy = 0 \iff x^2e^{-x^3}dx + \frac{y}{1+y^2}dy = 0$$

$$\therefore \int \frac{y}{1+y^2}dy = -\int x^2e^{-x^3}dx$$

$$\frac{1}{2}\ln(1+y^2) = \frac{1}{3}e^{-x^3} + c$$

$$y(0) = 0 \Rightarrow \frac{1}{2}\cancelto{0}{\ln(1)} = \frac{1}{3} + c \qquad \therefore c = -\frac{1}{3}$$

$$\frac{1}{2}\ln(1+y^2) = \frac{1}{3}e^{-x^3} - \frac{1}{3}$$

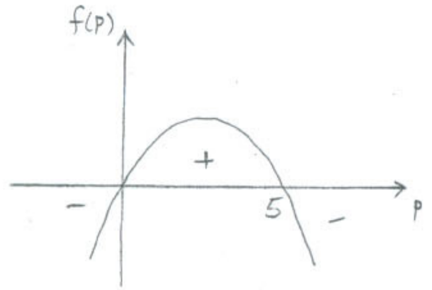
Answer: $\boxed{\frac{1}{2}\ln(1+y^2) = \frac{1}{3}e^{-x^3} - \frac{1}{3}}$

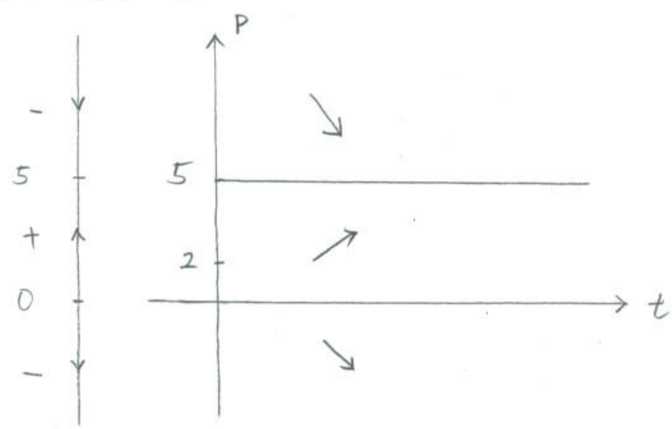
2. Using the phase line or direction field, find $\lim_{t\to\infty} p(t)$, where $p(t)$ is the solution to the IVP

$$\frac{dp}{dt} = 15p - 3p^2, \quad p(0) = 2.$$

$$\frac{dP}{dt} = -3P(P-5)$$

Let $f(P) = -3P(P-5)$




$$\therefore \lim_{t\to\infty} P(t) = 5, \text{ since } P(0) = 2$$

$\lim_{t\to\infty} p(t) =$ $\boxed{5}$