

Name: _____ Partners: _____

1. Find the maximum interval I, on which there exists a unique solution for the IVP:

$$\begin{cases} x'(t) = (t^2 + 1)x(t) - \sqrt{2t + 1}y(t) + 3t \\ y'(t) = \frac{1}{t-2}x(t) - y(t) + \ln(\cos t) \\ x(1) = 3, \quad y(1) = -1 \end{cases}$$

2. (a) Using eigenvalues and eigenvectors of the coefficient matrix, solve the following IVP,

$$\begin{cases} x'(t) = x(t) - 2y(t) \\ y'(t) = 2x(t) + y(t) \\ x(0) = 0, \quad y(0) = -3 \end{cases}$$

- (b) Using the matrix exponential function, solve the following IVP,

$$\begin{cases} x' = -x + 2y \\ y' = -y \\ x(0) = 3, \quad y(0) = -2. \end{cases}$$

- (c) Find
- $\lim_{t \rightarrow \infty} \frac{x(t)}{y(t)}$
- , where
- $x(t)$
- and
- $y(t)$
- satisfy the following IVP,

$$\begin{cases} x' = -x + 2y \\ y' = x - 2y \\ x(0) = 3, \quad y(0) = 5. \end{cases}$$

3. (a) Find
- $\lim_{t \rightarrow \infty} y(t)$
- , where
- $y(t)$
- is the solution of the following IVP,

$$\begin{cases} x' = x^2 + y^2 + 1 \\ y' = x \\ x(0) = 1, \quad y(0) = -2. \end{cases}$$

- (b) Find
- $x(5) + y(5)$
- , where
- $x(t)$
- and
- $y(t)$
- satisfy the following IVP,

$$\begin{cases} x' = 2x + y + 3 \\ y' = -2x - y - 1 \\ x(0) = 2, \quad y(0) = 1. \end{cases}$$

4. Use an exponential matrix to solve the following IVP,
- $\begin{cases} x' = 2x \\ y' = x + 2y \\ x(0) = -1, \quad y(0) = 1. \end{cases}$