MATH 205 **LAB 5** **Spring 2009**

Name : _____ Partners : _____

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1. Find a general solution to the following equations for $x > 0$. (Hint: Try with $y = x^m$.)
   (a) $x^2y''(x) + 2xy'(x) - 6y(x) = 0$
   (b) $x^3y'''(x) + x^2y''(x) - 2xy'(x) + 2y = 0$

2. Find the maximum interval of existence of a unique solution to the following IVP, and explain carefully your answer.

$$(\sin x)y'' + \frac{1}{(3x-22)(2x-1)}\,y' + (x^2+1)y = e^{2x}, \quad y'(8) = 0, \quad y(8) = -1$$

3. Consider the functions $y_1 = t$ and $y_2 = t^2$.
   (a) Are y_1 and y_2 linearly independent on the interval $(-1, 1)$? Explain.
   (b) Can $\{y_1, y_2\}$ be a fundamental set of solutions for a homogeneous second order linear DE defined on $(-1, 1)$? Explain.

4. Do #35 on page 168

5. Use the fact that $y_1(t) = \cos t - \sin 2t$ solves the IVP,

$$\begin{cases} y''(t) + y(t) = 3\sin 2t \\ y(0) = 1, \quad y'(0) = -2 \end{cases}$$

   and $y_2(t) = \frac{1}{2}e^{3t} + \sin t$ solves the IVP,

$$\begin{cases} y''(t) + y(t) = 5e^{3t} \\ y(0) = \dfrac{1}{2}, \quad y'(0) = \dfrac{5}{2}, \end{cases}$$

   to find the solution $y_3(t)$ to the following IVP,

$$\begin{cases} y''(t) + y(t) = 2\sin 2t - 2e^{3t} \\ y(0) = -\dfrac{6}{5}, \quad y'(0) = \dfrac{1}{15}. \end{cases}$$