Lab 5

1. $y=x^m,\ y'=mx^{m-1},\ y''=m(m-1)x^{m-2},\ \dots$

(a)
$$0 = x^2y''+2xy'-6y = [m(m-1)+2m-6]x^m = (m+3)(m-2)x^m \Rightarrow m=-3,2$$

$\therefore\ \underset{\sim}{y=c_1x^{-3}+c_2x^2}$

(b)
$$0=x^3y'''+x^2y''-2xy'+2y = [m(m-1)(m-2)+m(m-1)-2m+2]x^m$$

$\therefore\ m^3-2m^2-m+2=0$

$(m-1)(\underbrace{m^2-m-2}_{-2,\ +1})=0$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -1 & 2 \\ & & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

$(m-1)(m+1)(m-2)=0$

$\therefore\ m=-1,1,2$

$\therefore\ y=c_1x^{-1}+c_2x+c_3x^2$

2. $$y''+\frac{1}{(\sin x)(3x-22)(2x-1)}y' + \frac{x^2+1}{\sin x}y = \frac{e^{2x}}{\sin x},\quad x_0=8$$

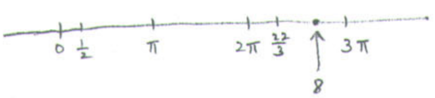
$\sin x=0 \iff x=n\pi\ (n\in\mathbb{Z})$

$3x-22=0 \iff x=\frac{22}{3}=7.33\dots$

$2x-1=0 \iff x=\frac{1}{2}$

$\therefore$ Interval of continuity of coefficient ftns & source ftn around $x=8$ is

$\left(\frac{22}{3},\ 3\pi\right)$



3. $y_1=t \quad y_2=t^2$

(a) Yes, y_1 & y_2 are lin indep

$$W[y_1,y_2](t)=\begin{vmatrix} t & t^2 \\ 1 & 2t\end{vmatrix} = 2t^2-t^2 = t^2$$

is not identically zero in $\mathbb{R}$

$\therefore\ y_1$ & y_2 are lin indep in $\mathbb{R}$

(b) No.

(If they were, $W[y_1,y_2](t)\neq 0,\ \forall\, t\in(-1,1)$, by Thm3 in sec 4.2. But $W[y_1,y_2](0)=0$, i.e., $\rightarrow\!\leftarrow$.)

4. (a) on $(0,\infty)$

$y_1(t)=t^3,\quad y_2(t)=t^3\equiv y_1(t)$

$\therefore$ linearly dependent

(b) on $(-\infty,0)$

$y_1(t)=t^3,\quad y_2(t)=-t^3\equiv -y_1(t)$

$\therefore$ linearly dependent

(c) on $(-\infty,\infty)$

Suppose $c_1t^3+c_2|t^3|=0,\ \forall t\in\mathbb{R}$

$$\left.\begin{array}{l} t=1 \to c_1+c_2=0 \\ t=-1 \to -c_1+c_2=0 \end{array}\right\} c_1=c_2=0$$

$\therefore$ linearly independent.

(d) $y_1(t)=t^3 \qquad y_1'(t)=3t^2$

$$y_2(t)=\begin{cases} t^3, & \text{if } t\geq 0 \\ -t^3, & \text{if } t<0\end{cases}\qquad y_2'(t)=\begin{cases} 3t^2, & \text{if } t\geq 0 \\ -3t^2, & \text{if } t<0\end{cases}$$

$$[W(y_1,y_2)](t)=\begin{cases}\begin{vmatrix} t^3 & t^3 \\ 3t^2 & 3t^2\end{vmatrix}, & \text{if } t\geq 0 \\ \begin{vmatrix} t^3 & -t^3 \\ 3t^2 & -3t^2\end{vmatrix}, & \text{if } t<0\end{cases}$$

$$=\begin{cases} 3t^5-3t^5, & \text{if } t\geq 0 \\ -3t^5+3t^5, & \text{if } t<0\end{cases}$$

$$=0,\quad \forall t\in\mathbb{R}$$

It does not contradict #34 (b), because there is no "evidence" that y_1 & y_2 are solutions to a linear 2nd order homogeneous eq on $(-\infty,\infty)$. In fact, the results in (c) & (d) & Thm3 in sec 4.2 implies that y_1 & y_2 cannot be solutions to a linear 2nd order homogeneous equation.

5. $y''+y=0$

$\to r^2+1=0 \to r=\pm i$

$\therefore\ \tilde{y}_1(t)=\cos t,\quad \tilde{y}_2(t)=\sin t$

$\therefore$ A general solution $y_i(t)$ to

$y''+y=f_i(t)$

is given by

$y_i(t)=y_{p_i}(t)+c_1\cos t+c_2\sin t$

Since $y_1=\cos t-\sin 2t$ solves

$y''+y=\underbrace{3\sin 2t}_{f_1(t)}$,

$\therefore\ y_{p_1}(t)=-\sin 2t$

Since $y_2(t)=\frac{1}{2}e^{3t}+\sin t$ solves

$y''+y=\underbrace{5e^{3t}}_{=f_2(t)}$,

$\therefore\ y_{p_2}(t)=\frac{1}{2}e^{3t}$

Note that

$$y''+y = 2\sin 2t-2e^{3t} = \frac{2}{3}(3\sin 2t)-\frac{2}{5}(5e^{3t}) = \frac{2}{3}f_1(t)-\frac{2}{5}f_2(t)$$

$$\therefore\ y_{p_3} = \frac{2}{3}y_{p_1}-\frac{2}{5}y_{p_2} = \frac{2}{3}(-\sin 2t)-\frac{2}{5}\left(\frac{1}{2}e^{3t}\right) = -\frac{2}{3}\sin 2t-\frac{1}{5}e^{3t}$$

$\therefore\ y_3(t)=-\frac{2}{3}\sin 2t-\frac{1}{5}e^{3t}+c_1\cos t+c_2\sin t$

$y_3'(t)=-\frac{4}{3}\cos 2t-\frac{3}{5}e^{3t}-c_1\sin t+c_2\cos t$

$-\frac{6}{5}=y_3(0)=-\frac{1}{5}+c_1 \Rightarrow c_1=-1$

$\frac{1}{15}=y_3'(0)=-\frac{4}{3}-\frac{3}{5}+c_2 = -\frac{29}{15}+c_2$

$\therefore\ c_2=\frac{1}{15}+\frac{29}{15}=\frac{30}{15}=2$

Thus,

$y_3(t)=-\frac{2}{3}\sin 2t-\frac{1}{5}e^{3t}-\cos t+2\sin t$