

Name : _____ Partners : _____

1. A tank initially contains 100 gallons of pure water. A brine solution containing 1 pound of salt per gallon enters the tank at the rate of 2 gallons per minute. The well-mixed solution exits the tank at the rate of 3 gallons per minute. Thus the tank is empty after 100 minutes. Find the amount of salt in the tank t minutes after the process begins for $0 \leq t \leq 100$, and find the time at which the tank contains the most salt.

2. Consider the population, $P(t)$ of a fruit fly at time t in a closed container. At time $t = 0$, its population was 1000. Suppose that its birth rate, $\beta(t)$ and its death rate, $\delta(t)$ at time t are given as

$$\beta(t) = 10 - 0.001P(t) , \quad \delta(t) = 2 + 0.001P(t).$$

Find $\lim_{t \rightarrow \infty} P(t)$.

3. Suppose that a community contains 15000 people who are susceptible to a spreading contagious disease. At time $t = 0$ the number $N(t)$ of people who have the disease is 5000 and is increasing by 500 per day. How long will it take for another 5000 people to contract the disease? Assume that $N'(t)$ is proportional to the product of the numbers of those who have the disease and those who do not.

4. Suppose that a lake is stocked with 10,000 fish one day and it is observed that the fish population is increasing at the rate of 20 fish per day initially. Let $P(t)$ denote the fish population in the lake t days later. Assuming that the birth rate is inversely proportional to \sqrt{P} and the death rate is always $\frac{2}{1000}$ (deaths per fish per day), find $P(t)$.

5. Suppose that when a certain lake is stocked with fish, the birth and death rates β and δ are both inversely proportional to \sqrt{P} .

(a) Show that

$$P(t) = \left(\frac{1}{2}kt + \sqrt{P_0} \right)^2.$$

(b) If $P_0 = 100$ and after 6 months there are 169 fish in the lake, how many will there be after 1 year?

6. The time rate of change of an alligator population P in a swamp is proportional to the square of P . The swamp contained a dozen alligators in 1988, two dozen in 1998.

(a) When will there be four dozen alligators in the swamp?

(b) What happens in the year 2008?