MATH 205 **LAB 2** **Spring 2009**

Name: ____________________________ Partners: ____________________________

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1. Solve the following IVP's.

   (a) $\dfrac{dx}{dt} = (x-2)(x-3), \;\; x(0) = 4$

   (b) $(1+y^2)\cos x\,dx = (1+\sin^2 x)y\,dy, \;\; y(\frac{\pi}{2}) = 3$

   (c) $\dfrac{dy}{dx} = x^2y + 1 + y + x^2, \;\; y(0) = 1$

   (d) $\dfrac{dy}{dx} = -\dfrac{3x^2+y^2}{2xy+3y^2}\,, \;\; y(2) = 1$

2. According to Newton's law of cooling, if an object at temperature T is immersed in a medium having the constant temperature M, then the rate of change of T is proportional to the difference of temperature $M - T$. This gives the differential equation $\frac{dT}{dt} = k(M-T)$, where k is a constant.

   (a) Solve the differential equation for T.

   (b) A thermometer reading 100°F is placed in a medium having a constant temperature of 70°F. After 6 min, the thermometer reads 80°F. What would be the reading after 20 min?

3. Consider the IVP: $\dfrac{\ln(x-1)}{x-3}\dfrac{dy}{dx} + y = \dfrac{1}{2-3x}, \;\; y(x_0) = y_0.$

   (a) Determine all possible regions in the xy-plane in which the IVP would have a unique solution through (x_0, y_0). Write these regions as subsets of the xy-plane and sketch their graphs.

   (b) Find the maximum possible interval of existence of solutions for the IVP if the initial condition is (i) $y(0) = 0$, (ii) $y(1.99) = 5$, (iii) $y(2.01) = -6$, (iv) $y(3.3) = 7$.

4. Suppose that the figure above is the graph of the solution $y = \eta(x)$ to the IVP,

$$\frac{dy}{dx} = f(y), \quad y(0) = 10.$$

   (a) Using the fact that the given DE is autonomous, sketch the direction field of the DE.

   (b) Let $y = \phi(x)$ be the function whose graph is obtained by translating the above solution curve horizontally by -3 (i.e., by 3 to the left). Is $\phi(x)$ also a solution to $\frac{dy}{dx} = f(y)$? Prove or disprove it.

   (c) Show that $\phi(x)$ defined in (b) is the solution to the IVP: $\frac{dy}{dx} = f(y), \;\; y(0) = 5$ and find the values of $\phi(4)$ and $\phi(8)$. (Hint: Use (b))

   (d) Let $\xi(x)$ be a solution to $\frac{dy}{dx} = g(y)$ and let $\phi_\alpha(x) = \xi(x - \alpha)$. Prove that $\phi_\alpha(x)$ is also a solution to $\frac{dy}{dx} = f(y)$, for all $\alpha \in \mathbb{R}$. What is the graphical meaning of this result?