Lab 1

1.(i) # 20 on P15

(a) $y'' + 6y' + 5y = 0$

$y \equiv e^{mx} \rightarrow y' = me^{mx}$

$y'' = m^2 e^{mx}$

$\therefore (m^2 + 6m + 5)e^{mx} = 0, \forall x$

$\Rightarrow m^2 + 6m + 5 = 0$

$\therefore m = -1, -5$

(b) $y^{(3)} + 3y'' + 2y' = 0$

$y \equiv e^{mx}$

$\rightarrow (m^3 + 3m^2 + 2m)e^{mx} = 0$

$\therefore m(m^2 + 3m + 2) = 0$

$\therefore m = 0, -1, -2$

(ii) # 21 on P15

(a) $3x^2y'' + 11xy' - 3y = 0$

$y \equiv x^m \rightarrow y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\therefore 3x^2 m(m-1)x^{m-2} + 11x\, m x^{m-1} - 3x^m = 0$

$\therefore [3m(m-1) + 11m - 3]\, x^m = 0$

$\therefore 3m^2 - 3m + 11m - 3 = 0$

$3m^2 + 8m - 3 = 0$

$\therefore m = -3, \frac{1}{3}$

(b) $x^2y'' - xy' - 5y = 0$

$y = x^m \rightarrow \cdots$

$\vdots$

$[m(m-1) - m - 5]\, x^m = 0$

$\therefore m^2 - 2m - 5 = 0$

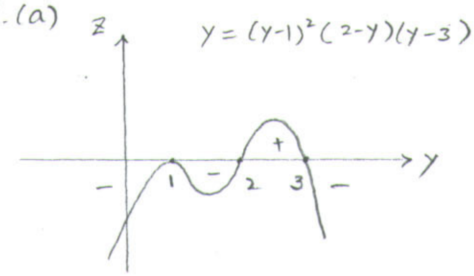
$\therefore m = \frac{2 \pm \sqrt{4 + 20}}{2}$

$= 1 \pm \frac{2\sqrt{6}}{2}$

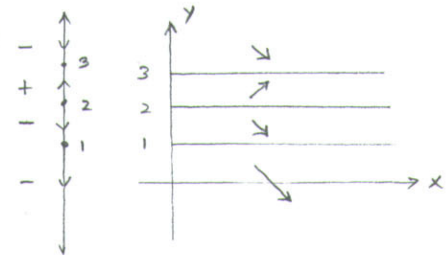
$= 1 \pm \sqrt{6}$

(iii) → back of the textbook

2.(a)



(b)



For the direction field, execute lab 1 - 2b.mws

(c)

α	0	1	1.5	2	2.5	3	4
$\lim_{x\to\infty} y(x)$	$-\infty$	1	1	2	3	3	3

3. (a) $y = \frac{1 + ce^{2x}}{1 - ce^{2x}}$ — ①

$\frac{dy}{dx} = \frac{2ce^{2x}(1 - ce^{2x}) - (1 + ce^{2x})(-2ce^{2x})}{(1 - ce^{2x})^2}$

$= \frac{2ce^{2x} - \cancel{2c^2e^{4x}} + \cancel{2c^2e^{4x}} + 2ce^{2x}}{(1 - ce^{2x})^2}$

$= \frac{4ce^{2x}}{(1 - ce^{2x})^2}$

$y^2 - 1 = \frac{(1 + ce^{2x})^2}{(1 - ce^{2x})^2} - \frac{(1 - ce^{2x})^2}{(1 - ce^{2x})^2}$

$= \frac{4ce^{2x}}{(1 - ce^{2x})^2}$

$\therefore \frac{dy}{dx} = y^2 - 1$

(b) $\frac{dy}{dx} = (y+1)(y-1)$

$\therefore y \equiv -1, y \equiv 1$ are solutions

If $c = 0$ in ①, $y \equiv 1$

However $y \equiv -1$ cannot be obtained from ①

$\left(\because \frac{1 + ce^{2x}}{1 - ce^{2x}} = -1 \right.$

$\Leftrightarrow 1 + \cancel{ce^{2x}} = -1 + \cancel{ce^{2x}}$

$\left. \Leftrightarrow 1 = -1 \quad \times \right)$

4.

$$\begin{cases} \frac{dy}{dx} = e^{x^2}y + \frac{y^2}{x^2+1} - (\sin x)y^3 \\ y(0) = 0 \end{cases} \quad \text{— } (*)$$

$f(x,y) = e^{x^2}y + \frac{y^2}{x^2+1} - (\sin x)y^3$

$\frac{\partial f}{\partial y} = e^{x^2} + \frac{2y}{x^2+1} - 3(\sin x)y^2$

$f(x,y)$ & $\frac{\partial f}{\partial y}$ are both conti on $\mathbb{R}^2$ (which contains $(0,0)$).

(i) By Thm 1, $\exists$ a unique solution to the IVP $(*)$ on an interval I around 0.

(ii) $\frac{dy}{dx} = y\left(e^{x^2} + \frac{y}{x^2+1} - (\sin x)y^2\right)$

$\therefore y \equiv 0$ is a solution to $(*)$

By (i) & (ii), $y \equiv 0$ is the unique solution to $(*)$.