2. $\lambda_1 = 3$, $\vec{u_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

17. (a) $\lambda_1 = 2$ $\quad \vec{u_1} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$

$\lambda_2 = -2$ $\quad \vec{u_2} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$

(b) $\vec{x}(t) = c_1 e^{2t} \vec{u_1} + c_2 e^{-2t} \vec{u_2}$

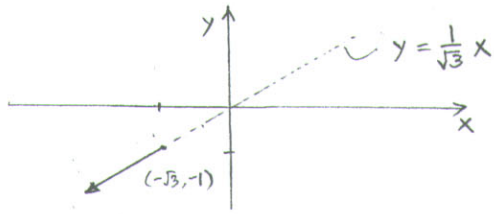
$-\vec{u_1} = \vec{x}(0) = c_1 \vec{u_1} + c_2 \vec{u_2}$

$\therefore (c_1 + 1)\vec{u_1} + c_2 \vec{u_2} = 0$

Since $\vec{u_1}$ & $\vec{u_2}$ are lin indep

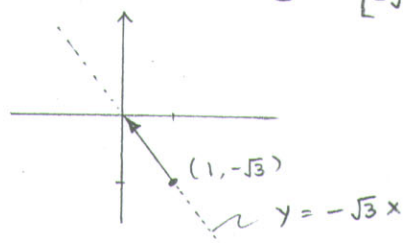
$c_1 = -1$, $c_2 = 0$

$\therefore \vec{x}(t) = -e^{2t} \vec{u_1} = e^{2t} \begin{bmatrix} -\sqrt{3} \\ -1 \end{bmatrix}$



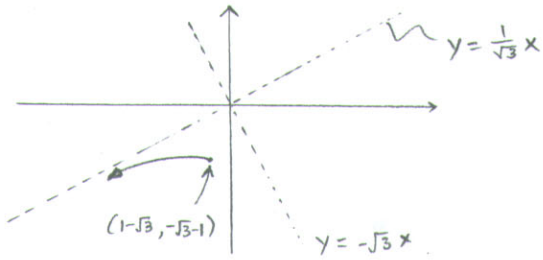
(c) By the similar way

$\vec{x}(t) = e^{-2t} \vec{u_2} = e^{-2t} \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$



(d) $\vec{x}(t) = -e^{2t} \vec{u_1} + e^{-2t} \vec{u_2}$

$= e^{2t} \begin{bmatrix} -\sqrt{3} \\ -1 \end{bmatrix} + e^{-2t} \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$

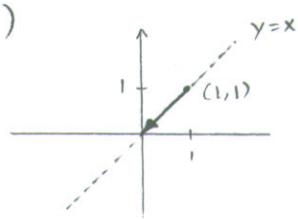


18 (a) $\lambda_1 = -1$, $\vec{u_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = -3$, $\vec{u_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

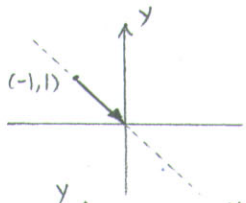
(b) $\vec{x} = e^{-t} \vec{u_1}$

$= e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

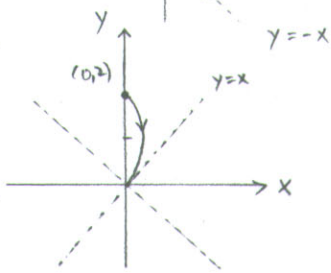


(c) $\vec{x} = -e^{-3t} \vec{u_2}$

$= e^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$



(d) $\vec{x} = e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$



20. $\begin{bmatrix} e^t & 4e^{4t} \\ -e^t & -e^{4t} \end{bmatrix}$

32. $\vec{x}' = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix} \vec{x}$ $\quad \vec{x}(0) = \begin{bmatrix} -10 \\ -6 \end{bmatrix}$

$0 = |A - \lambda I| = \begin{vmatrix} 6-\lambda & -3 \\ 2 & 1-\lambda \end{vmatrix} = (\lambda - 6)(\lambda - 1) + 6 = \lambda^2 - 7\lambda + 12$

-3
-4

$\therefore r = 3, 4$

(i) $r_1 = 3$: $\begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow 3a - 3b = 0$

$\therefore a - b = 0$

$a = 1 \rightarrow b = 1$ $\quad \therefore \vec{u_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(ii) $r_2 = 4$: $\begin{bmatrix} 2 & -3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \therefore 2a - 3b = 0$

$a = 3 \rightarrow b = 2$ $\quad \therefore \vec{u_2} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\therefore \vec{x}(t) = c_1 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -10 \\ -6 \end{bmatrix} = \vec{x}(0) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\therefore c_1 + 3c_2 = -10$

$-) \; c_1 + 2c_2 = -6$

$c_2 = -4$

$\rightarrow c_1 - 12 = -10$

$\therefore c_1 = 2$

$\therefore \vec{x}(t) = 2e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 4e^{4t} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2e^{3t} - 12e^{4t} \\ 2e^{3t} - 8e^{4t} \end{bmatrix}$