4. (a) $AB = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 12 & 4 \\ -1 & 8 & 4 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 3 & 2 \\ -1 & 15 \end{bmatrix}$

16. (a) $|A| = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 1\cdot\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} + 1\cdot\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix}$

$= 2(4-1) + (-2-1) + (-1-2)$

$= 6 - 3 - 3 = 0$

$\therefore$ A is singular

(b) $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ -1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{array}\right] \xrightarrow{[1]\leftrightarrow[3]} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ -1 & 2 & 1 & 1 \\ 2 & -1 & 1 & 3 \end{array}\right]$

$\xrightarrow[{[3]-2[1]}]{[2]+[1]} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 3 & 3 & 4 \\ 0 & -3 & -3 & -3 \end{array}\right] \xrightarrow{[3]+[2]} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 3 & 3 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\therefore \quad 0\cdot x_1 + 0x_2 + 0\cdot x_3 = 1$

i.e. $0 = 1 \quad \rightarrow\!\!\times$

$\therefore$ No solution

(c) $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ -1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 3 \end{array}\right] \xrightarrow{[1]\leftrightarrow[3]} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ -1 & 2 & 1 & 0 \\ 2 & -1 & 1 & 3 \end{array}\right]$

$\xrightarrow[{[3]-2[1]}]{[2]+[1]} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 3 & 3 & 3 \\ 0 & -3 & -3 & -3 \end{array}\right] \xrightarrow{[3]+[2]} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore \quad x_1 + x_3 = 2 \rightarrow x_1 = -x_3 + 2$

$\quad x_2 + x_3 = 1 \rightarrow x_2 = 1 - x_3$

$\therefore \; x_1 = -s+2, \quad x_2 = 1-s, \quad x_3 = s, \quad \forall s \in \mathbb{R}$

28. $|A - rI| = \begin{vmatrix} 3-r & 3 \\ 2 & 4-r \end{vmatrix} = 0$

$(3-r)(4-r) - 6 = 0$

$r^2 - 7r + 12 - 6 = 0$

$\therefore \quad r^2 - 7r + 6 = 0$

$-1$

$-6$

$(r-1)(r-6) = 0$

$\therefore r = 1, 6$

36. $\vec{x}(t) = \begin{bmatrix} 0 \\ e^t \\ -3e^t \end{bmatrix} \quad \therefore \vec{x}'(t) = \begin{bmatrix} 0 \\ e^t \\ -3e^t \end{bmatrix}$

$A\vec{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ e^t \\ -3e^t \end{bmatrix} = \begin{bmatrix} 0 \\ e^t \\ -3e^t \end{bmatrix} = \vec{x}'(t)$

$\therefore \begin{bmatrix} 0 \\ e^t \\ -3e^t \end{bmatrix}$ is a solution to

the eq $\vec{x}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\vec{x}$