HW 6.1

3. $P_1(x) = -1$, $P_2(x) = \sqrt{x-1}$, $f(x) = \tan x$

P_1 is conti on $\mathbb{R}$

P_2 is conti on $[1, \infty)$

f is conti on $\cdots \cup (\frac{\pi}{2}, \frac{3\pi}{2}) \cup (\frac{3\pi}{2}, \frac{5\pi}{2}) \cup \cdots$

$\therefore$ P_1, P_2, & f are conti on $(\frac{3\pi}{2}, \frac{5\pi}{2})$

& $t_0 = 5 \in (\frac{3\pi}{2}, \frac{5\pi}{2})$

$\therefore$ Answer is $(\frac{3\pi}{2}, \frac{5\pi}{2})$

4. $(-1, 0)$

6. $(0, 1)$

9. $\{\sin^2 x, \cos^2 x, 1\}$ on $(-\infty, \infty)$

Since $1\cdot\sin^2 x + 1\cdot\cos^2 x + (-1)\cdot 1 = 0$, $\forall x \in \mathbb{R}$

they are linearly indep on $\mathbb{R}$

14. lin. indep

16. $y''' - y'' + 4y' - 4y = 0$

ch. eq : $r^3 - r^2 + 4r - 4 = 0$

$\Leftrightarrow (r-1)(r^2+4) = 0$

$\therefore r = 1$, $\pm 2i$

(i) e^x, $\cos 2x$, $\sin 2x$ are solutions

(ii) $W[e^x, \cos 2x, \sin 2x] = \begin{vmatrix} e^x & \cos 2x & \sin 2x \\ e^x & -2\sin 2x & 2\cos 2x \\ e^x & -4\cos 2x & -4\sin 2x \end{vmatrix}$

$= e^x \begin{vmatrix} -2\sin 2x & 2\cos 2x \\ -4\cos 2x & -4\sin 2x \end{vmatrix} - \cos 2x \begin{vmatrix} e^x & 2\cos 2x \\ e^x & -4\sin 2x \end{vmatrix}$

$+ \sin 2x \begin{vmatrix} e^x & -2\sin 2x \\ e^x & -4\cos 2x \end{vmatrix}$

$= e^x(8\sin^2 2x + 8\cos^2 2x) - \cos 2x(-4e^x \sin 2x - 2e^x\cos 2x)$
$+ \sin 2x(-4e^x\cos 2x + 2e^x \sin 2x)$

$= 8e^x + \cancel{4e^x \cos 2x \sin 2x} + 2e^x\cos^2 2x$
$\cancel{-4e^x \cos 2x \sin 2x} + 2e^x \sin^2 2x$

$= 8e^x + 2e^x = 10e^x > 0$, $\forall x \in \mathbb{R}$

$\therefore$ e^x, $\cos 2x$, $\sin 2x$ are lin. indep

By (i) & (ii), $\{e^x, \cos 2x, \sin 2x\}$ are a fundamental solution set

20. (a) $y = c_1 + c_2 x + c_3 x^3 + x^2$

(b) $y = 2 - x^3 + x^2$

24. $L[y] = y''' - xy'' + 4y' - 3xy$

$y_1 = \cos 2x$ $\quad$ $y_2 = -\frac{1}{3}$

$y_1' = -2\sin 2x$, $y_1'' = -4\cos 2x$, $y_1''' = 8\sin 2x$

$y_2' = y_2'' = y_2''' = 0$

$\therefore L[y_1] = \cancel{8\sin 2x} + 4x\cos 2x \cancel{- 8\sin 2x} - 3x\cos 2x$
$= x\cos 2x \quad (\equiv f_1(x))$

$L[y_2] = 0 + 0 + 0 + x = x \quad (\equiv f_2(x))$

(a) $L[y] = 7x\cos 2x - 3x$
$= 7f_1(x) - 3f_2(x)$

$\therefore$ $y = 7y_1 - 3y_2 = 7\cos 2x + 1$

(b) $L[y] = -6x\cos 2x + 11x$
$= -6f_1(x) + 11f_2(x)$

$\therefore$ $y = -6y_1 + 11y_2 = -6\cos 2x - \frac{11}{3}$