205 HW 4.6 W08

2. $y'' + y = \sec t$

ch. eq : $r^2 + 1 = 0 \quad \therefore r = \pm i$

$y_h = c_1 \cos t + c_2 \sin t$

$y_p \equiv v_1 \cos t + v_2 \sin t$

$W[y_1, y_2] = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t$

$\equiv 1$

$v_1 = \int \frac{-\sec t \sin t}{1} dt = -\int \frac{\sin t}{\cos t} dt$

$= \ln|\cos t|$

$v_2 = \int \frac{\sec t \cos t}{1} dt = \int dt = t$

$\therefore y_p = \cos t \ln|\cos t| + t \sin t$

$\therefore y = c_1 \cos t + c_2 \sin t + \cos t \ln|\cos t| + t \sin t$

4. $y = c_1 e^t + c_2 e^{-t} - 2t - 4$

10. $y = c_1 e^{-2t} + c_2 t e^{-t} + \frac{1}{2} t^2 e^{-2t} \ln t$

$\boxed{-\frac{3}{4} t^2 e^{-2t}}$

22. $y'' - \frac{4}{t} y' + \frac{6}{t^2} y = t + \frac{1}{t^2}$ ; $y_p = v_1 t^2 + v_2 t^3$

$W = \begin{vmatrix} t^2 & t^3 \\ 2t & 3t^2 \end{vmatrix} = 3t^4 - 2t^4 = t^4$

$v_1 = \int \frac{-(t + \frac{1}{t^2}) t^3}{t^4} dt = \int (-1 - \frac{1}{t^3}) dt = -t + \frac{1}{2} \frac{1}{t^2}$

23. $t y'' + (5t - 1) y' - 5y = t^2 e^{-5t}$

$\therefore y'' + (5 - \frac{1}{t}) y' - \frac{5}{t} y = t e^{-5t}$

$y_1 = 5t - 1 \quad , \quad y_2 = e^{-5t}$

$y_p \equiv v_1 (5t - 1) + v_2 e^{-5t}$

$W[y_1, y_2] = \begin{vmatrix} 5t-1 & e^{-5t} \\ 5 & -5e^{-5t} \end{vmatrix}$

$= -25 t e^{-5t} + \cancel{5 e^{-5t}} - \cancel{5 e^{-5t}} = -25 t e^{-5t}$

$v_1 = \int \frac{-e^{-5t} \cancel{t e^{-5t}}}{-25 \cancel{t e^{-5t}}} dt = \frac{1}{25} \int e^{-5t} dt$

$= -\frac{1}{125} e^{-5t}$

$v_2 = \int \frac{(5t-1) \cancel{t e^{-5t}}}{-25 \cancel{t e^{-5t}}} dt = \int \left(-\frac{1}{5} t + \frac{1}{25}\right) dt$

$= -\frac{1}{10} t^2 + \frac{1}{25} t$

$\therefore y_p = -\frac{1}{125} e^{-5t} (5t - 1) + \left(-\frac{1}{10} t^2 + \frac{1}{25} t\right) e^{-5t}$

$= e^{-5t} \left(-\cancel{\frac{1}{25} t} + \frac{1}{125} - \frac{1}{10} t^2 + \cancel{\frac{1}{25} t}\right)$

$= \left(\frac{1}{125} - \frac{t^2}{10}\right) e^{-5t}$

$v_2 = \int \frac{(t + \frac{1}{t^2}) t^2}{t^4} dt = \int (\frac{1}{t} + \frac{1}{t^4}) dt = \ln|t| - \frac{1}{3} \frac{1}{t^3}$

$\therefore y_p = (-t + \frac{1}{2t^2}) t^2 + (\ln|t| - \frac{1}{3t^3}) t^3$

$= -t^3 + \frac{1}{2} + t^3 \ln|t| - \frac{1}{3}$

$= t^3 \ln|t| + \frac{1}{6} \underbrace{- t^3}_{\text{part of } y_h}$

$\therefore$ Choose $y_p = t^3 \ln|t| + \frac{1}{6}$