HW 4.5

2.

$y_1(t) = \frac{1}{4}\sin 2t \sim y'' + 2y' + 4y = \cos 2t$

$y_2(t) = \frac{t}{4} - \frac{1}{8} \sim y'' + 2y' + 4y = t$

$f_1(t) = \cos 2t \quad , \quad f_2(t) = t$

(a) $y'' + 2y' + 4y = t + \cos 2t$

$= f_1(t) + f_2(t)$

$\therefore y(t) = y_1(t) + y_2(t)$

$= \frac{1}{4}\sin 2t + \frac{t}{4} - \frac{1}{8}$

(b) $y'' + 2y' + 4y = 2t - 3\cos 2t$

$= -3f_1(t) + 2f_2(t)$

$\therefore y(t) = -3y_1(t) + 2y_2(t)$

$= -\frac{3}{4}\sin 2t + \frac{t}{2} - \frac{1}{4}$

(c) $y'' + 2y' + 4y = 11t - 12\cos 2t$

$= -12f_1(t) + 11f_2(t)$

$\therefore y(t) = -12y_1(t) + 11y_2(t)$

$= -3\sin 2t + \frac{11}{4}t - \frac{11}{8}$

4. $t + c_1 + c_2 e^{-t}$

7. $y'' - 2y' + y = 2e^x$

$y_p(x) = x^2 e^x$

ch. eq : $r^2 - 2r + 1 = 0$

$\therefore r = 1, 1$ (double root)

$\therefore y_h = c_1 e^x + c_2 x e^x$

$\therefore y = x^2 e^x + c_1 e^x + c_2 x e^x$

8. $\tan x + c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$