

HW 4.5

2.

$$y_1(t) = \frac{1}{4} \sin 2t \sim y'' + 2y' + 4y = \cos 2t$$

$$y_2(t) = \frac{t}{4} - \frac{1}{8} \sim y'' + 2y' + 4y = t$$

$$f_1(t) = \cos 2t, \quad f_2(t) = t$$

$$\begin{aligned} \text{(a)} \quad y'' + 2y' + 4y &= t + \cos 2t \\ &= f_1(t) + f_2(t) \end{aligned}$$

$$\begin{aligned} \therefore y(t) &= y_1(t) + y_2(t) \\ &= \frac{1}{4} \sin 2t + \frac{t}{4} - \frac{1}{8} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad y'' + 2y' + 4y &= 2t - 3 \cos 2t \\ &= -3f_1(t) + 2f_2(t) \end{aligned}$$

$$\begin{aligned} \therefore y(t) &= -3y_1(t) + 2y_2(t) \\ &= -\frac{3}{4} \sin 2t + \frac{t}{2} - \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad y'' + 2y' + 4y &= 11t - 12 \cos 2t \\ &= -12f_1(t) + 11f_2(t) \end{aligned}$$

$$\begin{aligned} \therefore y(t) &= -12y_1(t) + 11y_2(t) \\ &= -3 \sin 2t + \frac{11}{4}t - \frac{11}{8} \end{aligned}$$

$$4. \quad t + c_1 + c_2 e^{-t}$$

$$7. \quad y'' - 2y' + y = 2e^x$$

$$y_p(x) = x^2 e^x$$

$$\text{ch. eq: } r^2 - 2r + 1 = 0$$

$$\therefore r = 1, 1 \text{ (double root)}$$

$$\therefore y_h = c_1 e^x + c_2 x e^x$$

$$\therefore y = x^2 e^x + c_1 e^x + c_2 x e^x$$

$$8. \quad \tan x + c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$$