2. $y'' + y = 0$

ch. eq : $r^2 + 1 = 0$

$\therefore\ r = \pm i \quad (\alpha = 0,\ \beta = 1)$

$\therefore\ y(t) = c_1 \cos t + c_2 \sin t$

18. $c_1 e^{-7t} + c_2 e^{\frac{t}{2}}$

26. $e^t - 3te^t$

32. (a) $10y'' + 250y = 0$

$\therefore \begin{cases} y'' + 25y = 0 \\ y(0) = 0.3\ ,\quad y'(0) = -0.1 \end{cases}$

ch. eq : $r^2 + 25 = 0$

$\therefore\ r = \pm 5i$

$y(t) = c_1 \cos 5t + c_2 \sin 5t$

$y'(t) = -5c_1 \sin 5t + 5c_2 \cos 5t$

$0.3 = y(0) = c_1$

$-0.1 = y'(0) = 5c_2 \Rightarrow c_2 = -0.02$

$\therefore\ y(t) = 0.3 \cos 5t - 0.02 \sin 5t$

(b) $\frac{\beta}{2\pi} = \frac{5}{2\pi}$

33. (a) $10y'' + 60y' + 250y = 0$

$\begin{cases} y'' + 6y' + 25y = 0 \\ y(0) = 0.3\ ,\quad y'(0) = -0.1 \end{cases}$

ch. eq : $r^2 + 6r + 25 = 0$

$\therefore\ r = -3 \pm 4i$

$\therefore\ y(t) = e^{-3t}(c_1 \cos 4t + c_2 \sin 4t)$

$y'(t) = -3e^{-3t}(c_1 \cos 4t + c_2 \sin 4t)$

$\quad + e^{-3t}(-4c_1 \sin 4t + 4c_2 \cos 4t)$

$0.3 = y(0) = c_1$

$-0.1 = y'(0) = -3c_1 + 4c_2 = -0.9 + 4c_2$

$\therefore\ 4c_2 = 0.8 \qquad \therefore\ c_2 = 0.2$

$\therefore\ y(t) = 0.3e^{-3t} \cos 4t + 0.2e^{-3t} \sin 4t$

(b) $\frac{\beta}{2\pi} = \frac{4}{2\pi} = \frac{2}{\pi}$

(c) The introduction of a damping term lowers the frequency of the oscillation $\left(\frac{5}{2\pi} \to \frac{2}{\pi}\right)$. Also, it causes energy loss so that

$$\lim_{t\to\infty} y(t) = 0$$

$$\left(\because \lim_{t\to\infty} e^{-3t} = 0\right)$$