4. $c_1 e^{-3t} + c_2 t e^{-3t}$

8. $c_1 e^{\frac{t}{2}} + c_2 e^{-\frac{2t}{3}}$

15. ch. eq : $r^2 + 2r + 1 = 0$

$(r+1) = 0$

$\therefore r = -1, -1$ (double root)

$\therefore y(t) = c_1 e^{-t} + c_2 t e^{-t}$

$y'(t) = -c_1 e^{-t} + c_2 e^{-t} - c_2 t e^{-t}$

$1 = y(0) = c_1$

$-3 = y'(0) = -c_1 + c_2 = -1 + c_2$

$\therefore c_2 = -2$

$\therefore \underline{y(t) = e^{-t} - 2te^{-t}}$

28. Suppose $c_1 e^{3t} + c_2 e^{-4t} = 0, \ \forall t \in \mathbb{R}$ ①

$t = 0 \rightarrow c_1 + c_2 = 0 \Rightarrow c_2 = -c_1$

Differentiate ① to get

$3c_1 e^{3t} - 4c_2 e^{-4t} = 0, \ \forall t \in \mathbb{R}$

$t = 0 \rightarrow 3c_1 - 4c_2 = 0$

$c_2 = -c_1 \rightarrow 7c_1 = 0 \rightarrow c_1 = 0 \rightarrow c_2 = 0$

$\therefore c_1 = c_2 = 0$

$\therefore e^{3t}$ & e^{-4t} are lin. indep.

(2nd method to # 28)

$$W[e^{3t}, e^{-4t}] = \begin{vmatrix} e^{3t} & e^{-4t} \\ 3e^{3t} & -4e^{-4t} \end{vmatrix}$$

$= -4e^{3t}e^{-4t} - 3e^{3t}e^{-4t}$

$= -7e^{-t} \neq 0, \ \forall t \in \mathbb{R}$

$\therefore e^{3t}$ & e^{-4t} are lin. indep.

32. linearly dependent

($\because y_1(t) \equiv 0 \cdot y_2(t)$).