HW 3.4

3. $m = 500$, $v_0 = 0$, $g = 9.81$, $b = 50$

Use (6) on P111 to get

$$x(t) = \frac{500(9.81)}{50}t + \frac{500}{50}\left(0 - \frac{500(9.81)}{50}\right)\left(1 - e^{-\frac{50t}{500}}\right)$$

$$\therefore x(t) = 98.1t + 980e^{-\frac{t}{10}} - 981$$

$$x(t) = 1000$$

$$\rightarrow 98.1t + 980e^{-\frac{t}{10}} - 981 = 1000$$

$$\therefore 98.1t + 980e^{-\frac{t}{10}} = 1981 - (*)$$

If we ignore $980e^{-\frac{t}{10}}$, then we get $t \approx 20.2$. However, $980e^{-\frac{20.2}{10}} \approx 132.8$ is not negligible.

$\therefore$ Use either calculator or CAS to solve (*) to get

$$t \approx 18.64 \text{ (sec)}$$

7. Let t_* be the time when the chute opens

Let $x_1(t)$ denote the distance for $0 \le t \le t_*$

Using $m = 75$, $b_1 = 30$, $g = 9.81$, $v_0 = 0$, (5) & (6) on P114, we get

$$x_1(t) = \frac{75(9.81)}{30}t + \frac{75}{30}\left(0 - \frac{75(9.81)}{30}\right)\left(1 - e^{-\frac{30t}{75}}\right)$$

$$v_1(t) = \frac{75(9.81)}{30} + \left(0 - \frac{75(9.81)}{30}\right)e^{-\frac{30t}{75}}$$

$$v_1(t_*) = \frac{5(9.81)}{2} - \frac{5(9.81)}{2}e^{-\frac{2t_*}{5}} \equiv 20$$

$$\therefore \frac{5(9.81)}{2}e^{-\frac{2t_*}{5}} = \frac{5(9.81)}{2} - 20$$

$$e^{-\frac{2t_*}{5}} \approx 0.1845$$

$$\therefore t_* = -\frac{5}{2}\ln(0.1845) \approx 4.225 \text{ (sec)}$$

$$x_1(t_*) = \frac{5(9.81)}{2}(4.225) - \frac{25(9.81)}{4}\left(1 - e^{-\frac{2}{5}(4.225)}\right)$$

$$\approx 53.62 \text{ (m)}$$

$\therefore$ She is $2000 - 53.62 \approx 1946.38$ m above the ground when the chute opened

Let $\tau \equiv t - t_*$ (time in sec since the chute opened)

& $x_2(\tau)$ be the distance traveled τ secs after the chute open.

The $x_2(\tau)$ satisfies (6) on P114 with

$m = 75$, $b_2 = 90$, $g = 9.81$, $V_0 = 20$

$$\therefore x(\tau) = \frac{75(9.81)}{90}t + \frac{75}{90}\left(20 - \frac{75(9.81)}{90}\right)\left(1 - e^{-\frac{90t}{75}}\right)$$

$$\approx 8.175t + 9.8542 - 9.8542e^{-\frac{6t}{5}}$$

Solve $x(\tau) = 1946.38$

$$\therefore 8.175\tau + 9.8542 - 9.8542e^{-\frac{5\tau}{6}} = 1946.38$$

$$8.175\tau - 9.8542e^{-\frac{5\tau}{6}} = 1936.5258$$

Neglecting $-9.8542e^{-\frac{5\tau}{6}}$, we get

$$\tau \approx 236.9$$

Indeed, $\left|-9.8542e^{-\frac{5(236.9)}{6}}\right| \sim 1.8 \times 10^{-85}$

$$\therefore \tau = 236.9$$

$$\therefore t = t_* + \tau \approx 241.1 \text{ (sec)}$$