3. $r_i = 6$, $C_i = 0.2$, $r_o = 8$

$x(0) = 200 \cdot (0.005) = 1$

$V(t) = 200 - 2t$

$$\therefore \left[\begin{aligned} \frac{dx}{dt} &= 6 \cdot (0.2) - 8 \cdot \frac{x(t)}{200-2t} \\ &= 1.2 - \frac{4}{100-t} \cdot x \\ x(0) &= 1 \end{aligned}\right.$$

$$\therefore x' + \frac{4}{100-t} x = 1.2$$

$$\mu(t) = e^{\int \frac{4dt}{100-t}} = e^{-4\ln(100-t)} = (100-t)^{-4}$$

$$\therefore \frac{d}{dt}\left[(100-t)^{-4} x\right] = 1.2(100-t)^{-4}$$

$$(100-t)^{-4} x = 0.4(100-t)^{-3} + C$$

$$x(t) = 0.4(100-t) + C(100-t)^4$$

$$x(0) = 1 \rightarrow 1 = 40 + C(100)^4$$

$$\therefore C = -\frac{39}{10^8} = -3.9 \times 10^{-7}$$

$$\therefore \underline{x(t) = 0.4(100-t) - 3.9 \times 10^{-7}(100-t)^4}$$

$$C(t) = \frac{x(t)}{V(t)} = \frac{x(t)}{2(100-t)}$$

$$= 0.2 - 1.95 \times 10^{-7}(100-t)^3 \equiv 0.1$$

$$\Rightarrow 0.1 = 1.95 \times 10^{-7}(100-t)^3$$

$$(100-t)^3 = \frac{0.1}{1.95} \cdot 10^7 = \frac{10^6}{1.95}$$

$$\therefore 100 - t = \left(\frac{10^6}{1.95}\right)^{\frac{1}{3}}$$

$$\therefore \underline{t = 100 - \left(\frac{10^6}{1.95}\right)^{\frac{1}{3}} \approx 19.96 \text{ (min)}}$$

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2. $x(t) = 2.5 - 2e^{-\frac{3t}{25}}$

$t \approx 5.78$ (min)

A. $$\int \frac{dx}{x(7-x)} = \int 4\,dt \rightarrow \frac{1}{7}\int\left(\frac{1}{x} - \frac{1}{x-7}\right)dx = 4t + C$$

$$\therefore \ln \frac{x}{x-7} = 28t + C \rightarrow \frac{x}{x-7} = A \cdot e^{28t}$$

$$x(0) = 11 \rightarrow \frac{11}{4} = A \qquad \therefore \frac{x}{x-7} = \frac{11}{4}e^{28t}$$

$$\therefore \underline{x(t) = \frac{-\frac{77}{4}e^{28t}}{1 - \frac{11}{4}e^{28t}} = \frac{77}{11 - 4e^{-28t}}}$$

B. $$\frac{dx}{dt} = 0.8x - 0.004x^2$$

$$= 0.004x(200 - x)$$

(a) $k = 0.004$, $M = 200$

$\therefore$ Maximum : $\underline{200 \text{ (grams)}}$

(b) $x_0 = 50$

$$\therefore x(t) = \frac{M x_0}{x_0 + (M - x_0)e^{-kMt}}$$

$$= \frac{10^4}{50 + 150e^{-0.8t}}$$

$$x(t) = 100 \rightarrow 100 = \frac{10^4}{50 + 150e^{-0.8t}}$$

$$\therefore 50 + 150e^{-0.8t} = 100$$

$$150e^{-0.8t} = 50$$

$$e^{-0.8t} = \frac{1}{3}$$

$$-0.8t = -\ln 3$$

$$\therefore t = \frac{\ln 3}{0.8} \approx 1.37 \text{ (sec)}$$