HW Sec 2.6

2. homogeneous

4. Bernoulli

8. $y' = G(ax+by+c)$ type

18. $\frac{dy}{dx} = (x+y+2)^2$

$z \equiv x+y+2 \quad \therefore \frac{dz}{dx} = 1 + \frac{dy}{dx}$

$\therefore \frac{dz}{dx} - 1 = z^2 \rightarrow \frac{dz}{dx} = z^2+1$

$\int \frac{dz}{z^2+1} = \int dx$

$\therefore \tan^{-1} z = x + C$

$\therefore z = \tan(x+C)$

$\therefore x+y+2 = \tan(x+C)$

$\therefore \boxed{y = -x-2+\tan(x+C)}$

21. $\frac{dy}{dx} + \frac{1}{x}y = x^2y^2 \quad — (*)$

Bernoulli eq with $n=2$

$\therefore u \equiv y^{1-2} = y^{-1}$

$\therefore \frac{du}{dx} = -y^{-2}\frac{dy}{dx}$

Multipling $(*)$ by y^{-2} to get

$y^{-2}\frac{dy}{dx} + \frac{1}{x}y^{-1} = x^2$

$\therefore -\frac{du}{dx} + \frac{1}{x}u = x^2$

$\frac{du}{dx} - \frac{1}{x}u = -x^2$

$\mu(x) = e^{-\int \frac{dx}{x}} = e^{-\ln|x|} = \frac{1}{|x|}$

Choose $\mu(x) = \frac{1}{x}$

$\therefore \frac{d}{dx}\left[\frac{1}{x}u\right] = -x$

$\therefore \frac{1}{x}u = -\frac{1}{2}x^2 + C$

$u = -\frac{1}{2}x^3 + Cx$

$\therefore y^{-1} = -\frac{1}{2}x^3 + Cx$

$\therefore y = \frac{1}{-\frac{1}{2}x^3 + Cx}$

or $\boxed{y = \frac{2}{Cx - x^3}}$ $\quad 2C \rightarrow C$

22. $y^{-2} = -\frac{1}{2}e^{2x} + Ce^{-2x}$

or $y \equiv 0$